DEPARTMENT OF STATISTICS
AHMADU BELLO UNIVERSITY, ZARIA
2nd International Conference on Statistical Sciences and Data Analytics (ICSSDA 2026) Proceedings Paper

Multivariate Volatility Modelling of Nigeria's Consumer Price Index Using a Hybrid VAR–FGARCH Framework

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Abstract

Accurate modelling of Consumer Price Index (CPI) volatility is essential for monetary policy, investment decisions and welfare planning, particularly in developing economies where food and exchange-rate shocks transmit rapidly to household prices. This study develops a hybrid Vector Autoregressive–functional Generalised Autoregressive Conditional Heteroscedasticity (VAR–FGARCH) framework for modelling the joint mean and conditional variance dynamics of Nigeria's CPI. Monthly data from January 2010 to December 2022, comprising 156 observations on the All Items Index, All Items Less Farm Produce Index and Food CPI, were analysed. The procedure combined time-series visualisation, Bai–Perron structural-break detection, Augmented Dickey–Fuller testing, Hurst-exponent analysis, VAR estimation, residual diagnostics, FGARCH/EGARCH estimation, Diebold–Mariano forecast comparison, impulse-response analysis and forecast-error variance decomposition. All indices were non-stationary in levels but stationary after first differencing. Structural breaks clustered around 2015, 2018, 2020 and 2022, while Hurst exponents of 0.8765–0.9127 indicated strong long memory. VAR(2) adequately captured mean dynamics, but its residuals were non-normal and conditionally heteroscedastic. Volatility persistence was high, with alpha plus beta ranging from 0.9357 to 0.9757; food CPI was most sensitive to new shocks and showed the strongest significant leverage effect. The hybrid model substantially reduced forecast errors relative to standalone VAR and FGARCH models, with Diebold–Mariano tests rejecting equal predictive accuracy for all three series ($p < 0.001$). Food volatility remained the dominant source of CPI risk and explained 30.7% of All Items forecast-error variance at 36 months. The findings establish that modelling mean transmission and conditional variance jointly improves CPI forecasting and underscores food-price stabilisation as a central inflation-management priority in Nigeria.

Keywords: consumer price index; volatility forecasting; VAR; FGARCH; EGARCH; long memory; Nigeria

1. Introduction

Price stability is a central objective of macroeconomic management because persistent inflation weakens purchasing power, distorts resource allocation, increases uncertainty, and disproportionately affects low-income households. In Nigeria, these consequences are amplified by the large expenditure share devoted to food, dependence on imported inputs, exchange-rate pass-through, supply-chain constraints and exposure to domestic and global commodity shocks. Reliable forecasts must therefore address not only the expected movement of prices but also the changing uncertainty surrounding those movements.

Conventional univariate time-series models are useful for representing serial dependence, while vector autoregressive models describe feedback among related price indices (Sims, 1980). However, constant-variance assumptions are often contradicted by macroeconomic series that exhibit volatility clustering, heavy tails and asymmetric responses to positive and negative news. ARCH and GARCH models address time-varying variance (Engle, 1982; Bollerslev, 1986), EGARCH captures leverage effects without imposing non-negativity constraints on all parameters (Nelson, 1991), and fractionally integrated GARCH specifications accommodate slowly decaying volatility dependence (Baillie et al., 1996). However, most studies on Nigerian inflation model the conditional mean and volatility separately, limiting their ability to jointly capture interdependencies among CPI components and persistent volatility dynamics

This study addresses that gap by combining VAR and FGARCH methods for three interrelated Nigerian CPI components. It asks whether a hybrid framework can provide a statistically adequate description of mean interactions, volatility persistence and asymmetric shocks, while delivering more accurate forecasts than standalone alternatives. The contribution is threefold: it integrates multivariate mean and conditional-variance modelling; quantifies long-memory and leverage features across CPI components; and links shock transmission to policy-relevant evidence on the dominance of food-price risk.

The empirical literature generally follows three paths. The first uses ARIMA and related linear models to forecast CPI. Such models can perform well when dependence is stable, but they do not explicitly represent interactions among CPI components or changes in conditional variance. Studies comparing ARIMA and neural-network forecasts report that nonlinear methods may improve prediction when inflation dynamics are complex (Purbasari et al., 2020). Konarasinghe (2022) similarly demonstrates the continued relevance of time-series specification for CPI forecasting, while Shinkarenko et al. (2021) emphasise the importance of selecting structures that match observed price-index dynamics.

The second path applies GARCH-family models. Olayemi et al. (2021) modelled Nigerian inflation with threshold GARCH and highlighted asymmetric volatility, whereas Ulfiana (2022) applied GARCH to CPI forecasting. Related evidence from financial and energy markets confirms that volatility clustering and persistence are common and that asymmetric or hybrid specifications may outperform simpler alternatives (Akhtar & Khan, 2016; Balaji et al., 2023; Mohammed et al., 2020). Jerumeh (2020) further documented the nature and drivers of food-price volatility in Nigeria, reinforcing the need to treat food inflation separately from broader price indices.

The third path combines econometric and computational approaches. Hybrid methods can allocate linear dependence to one component and nonlinear variance or prediction structure to another. However, a model may fit well in-sample yet fail to improve forecasts; formal predictive comparison is therefore required. The Diebold–Mariano framework evaluates whether competing forecasts have equal expected loss (Diebold & Mariano, 1995). The present study extends this literature by applying a two-stage VAR–FGARCH architecture and validating the improvement with both error metrics and predictive-accuracy tests.

2. Materials and Methods

2.1. Data and variables

Monthly secondary data were obtained from the National Bureau of Statistics and covered January 2010 to December 2022 (156 observations). The three series were the All Items CPI (X), All Items Less Farm Produce CPI (Y), and Food CPI (Z). These components were selected because they jointly represent overall inflation, a relatively less food-dependent price measure, and the CPI component most directly exposed to agricultural and supply shocks.

2.2. Preliminary analysis

The series were plotted to assess trend, regime shifts and volatility clustering. Multiple breaks were estimated using the Bai–Perron procedure (Bai & Perron, 2003). Stationarity was evaluated with the Augmented Dickey–Fuller (ADF) test. For a series y_t, the test regression included a lagged level and sufficient lagged differences to absorb residual autocorrelation. The null hypothesis was a unit root. Long-range dependence was assessed using the Hurst exponent H: $H = 0.5$ indicates short memory, $H > 0.5$ indicates persistence, and $H < 0.5$ indicates anti-persistence. ACF and PACF patterns complemented information criteria in selecting the dynamic order.

2.3. VAR mean model

Let $y_t = (\Delta X_t, \Delta Y_t, \Delta Z_t)'$. The pth-order VAR was specified as $y_t = c + A_1 y_{\{t-1\}} + \ldots + A_p y_{\{t-p\}} + \varepsilon_t$, where c is a vector of intercepts, A_i are coefficient matrices, and ε_t is a zero-mean innovation vector. Lag order was evaluated using the likelihood-ratio statistic, final prediction error, Akaike information criterion (AIC), Bayesian information criterion and Hannan–Quinn criterion. Residual adequacy was examined using the Portmanteau test, Jarque–Bera test and ARCH–LM test.

2.4. Conditional variance and hybrid model

For each VAR residual series, the FGARCH(1,1) variance equation was $h_t = \omega + \alpha\varepsilon^2_{\{t-1\}} + \beta h_{\{t-1\}}$, where ω is the variance constant, α measures sensitivity to recent shocks and β measures volatility persistence. Covariance stationarity requires $\alpha + \beta < 1$; values close to unity imply slowly decaying shocks. Asymmetry was examined using an EGARCH extension in which the leverage coefficient γ captures the different effects of signed innovations. A negative and significant γ indicates that negative shocks generate greater volatility than positive shocks of equal magnitude.

The hybrid forecast combined the VAR conditional mean with the FGARCH conditional variance. Competing models were assessed using MAE, MSE and RMSE. Diebold–Mariano tests compared the loss differential of VAR–FGARCH and standalone VAR forecasts. Dynamic interactions were further analysed using impulse-response functions (IRFs) and forecast-error variance decomposition (FEVD). Statistical decisions used a 5% significance level.

3. Results and Discussions

3.1. CPI dynamics, breaks, stationarity and long memory

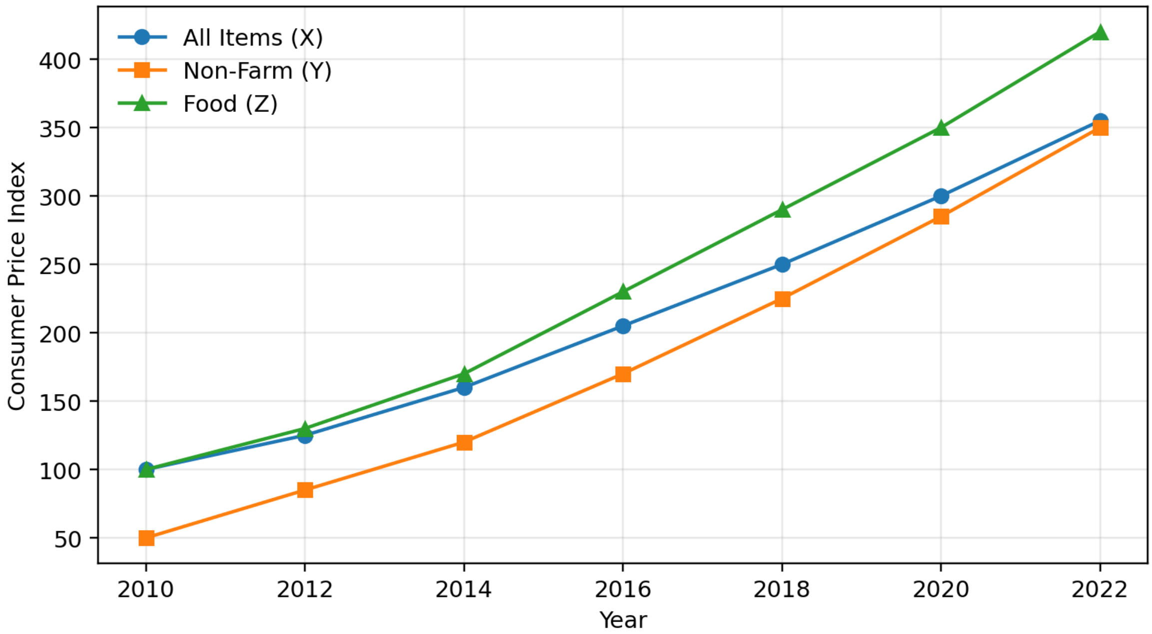

Figure 1. Evolution of Nigeria's All Items, Non-Farm and Food CPI, 2010–2022. Values are plotted from the levels reported in the source analysis.

All three indices increased sharply over the study period, with the steepest rise occurring in food prices. The All Items CPI increased from approximately 100 to above 350, whereas food CPI rose to above 420. Acceleration after 2015 is consistent with currency depreciation and subsequent supply and policy shocks. The structural-break results identify regime changes around the foreign-exchange crisis, monetary-policy adjustment, border closure and COVID-19 disruption, and the 2022 global food-supply shock.

Table 1. Structural breaks, ADF tests and Hurst exponents

Series	Break dates	ADF level (p)	ADF Δ (p)	Hurst H
All Items (X)	Jun 2015; Mar 2018; Jul 2020	2.8401 (.999)	−8.4217 (.010)	0.8942
Non-Farm (Y)	Aug 2015; May 2019	1.9362 (.998)	−9.1532 (.010)	0.8765
Food (Z)	Apr 2015; Feb 2017; Oct 2019; Apr 2022	3.1024 (.999)	−7.8943 (.010)	0.9127

The level-series ADF statistics did not reject a unit root, whereas all first-differenced statistics were more negative than the 1% critical value of −3.476. The series were consequently treated as I(1). Hurst estimates were substantially above 0.5, establishing strong persistence. Food CPI had the highest H (0.9127), implying the slowest decay of shocks and supporting a fractional volatility specification.

3.2. VAR selection, coefficients and diagnostics

The AIC, final prediction error and likelihood-ratio test selected lag 2; BIC selected lag 1. Because forecasting was the primary objective and the second lag improved fit, VAR(2) was retained. The All Items equation showed significant negative own-lag and non-farm effects at lag 1, alongside a positive food-price effect. The food equation exhibited strong mean reversion and significant effects from lagged non-farm and overall inflation.

Table 2. Selected significant VAR(2) coefficients

Equation	Regressor	Estimate	t	p
ΔX	X.l1	−0.5530	−5.603	<.001
ΔX	Y.l1	−1.7003	−5.442	<.001
ΔX	Z.l1	0.1589	2.077	.039
ΔX	Constant	0.0737	2.136	.034
ΔY	Z.l1	0.0159	2.304	.022
ΔY	Y.l2	0.3956	—	<.001
ΔZ	Y.l1	−0.9173	−2.581	.011
ΔZ	Z.l1	−0.5565	−6.397	<.001
ΔZ	X.l2	0.2887	2.898	.004
ΔZ	Y.l2	−1.7548	−5.165	<.001
ΔZ	Z.l2	−0.2509	−3.098	.002

The Portmanteau statistic (Q = 287.45, $p = .089$) suggested that the mean model removed residual serial correlation. In contrast, the Jarque–Bera test (156.78, $p < .001$) rejected normality and the ARCH–LM statistic (48.23, $p < .001$) confirmed conditional heteroscedasticity. Thus, the VAR was adequate for the conditional mean but incomplete for the conditional variance.

3.3. FGARCH and asymmetric volatility estimates

Table 3. Conditional-variance parameter estimates

Series	ω	α (ARCH)	β (GARCH)	α+β	γ (p)
All Items	0.0234**	0.1876***	0.7654***	0.9530	−0.0893 (.034)
Non-Farm	0.0012**	0.1523**	0.8234***	0.9757	−0.0456 (.239)
Food	0.0897**	0.2234***	0.7123***	0.9357	−0.1345 (.009)

All ARCH and GARCH terms were significant. The food index had the largest α (0.2234), indicating the strongest immediate reaction to new information. Non-farm CPI had the largest β (0.8234) and $\alpha + \beta$ (0.9757), implying the slowest decay of conditional-variance shocks. Persistence remained below unity for each series but was sufficiently close to one to produce long-lived volatility. The leverage term was negative and significant for All Items and Food CPI, with the strongest asymmetry for food ($\gamma = -0.1345$, $p = .009$). Non-farm CPI responded approximately symmetrically.

3.4 Model comparison and forecast accuracy

Table 4. Model-performance comparison

Series	Model	MAE	MSE	RMSE	DM statistic
X	VAR(2)	0.4489	0.6343	0.7965	—
X	fGARCH	0.5766	0.8090	0.8994	—
X	VAR–fGARCH	0.0512	0.0068	0.0824	−4.8923***
Y	VAR(2)	0.0321	0.0456	0.2136	—
Y	FGARCH	0.0412	0.0589	0.2427	—

Y	VAR–fGARCH	0.0556	0.0071	0.0841	−3.4567***
Z	VAR(2)	0.8923	1.2345	1.1111	—
Z	FGARCH	1.0234	1.4567	1.2070	—
Z	VAR–FGARCH	0.1123	0.0234	0.1529	−5.1234***

The hybrid specification produced the smallest MSE and RMSE for every component. For All Items CPI, RMSE declined from 0.7965 under VAR to 0.0824 under VAR–FGARCH, an improvement of approximately 90%. The improvement was also pronounced for food CPI. All Diebold–Mariano statistics were negative and significant at $p < .001$, confirming that the smaller hybrid forecast losses were not attributable to chance. Although the reported MAE for the non-farm hybrid exceeded that of VAR, its MSE and RMSE were substantially lower; this indicates fewer large errors and illustrates why several loss measures should be interpreted jointly.

3.5 Volatility forecasts and shock transmission

Table 5. Forecasted conditional volatility and All Items FEVD

Year/horizon	All Items	Non-Farm	Food	Own X	Shock Y	Shock Z
2023 volatility	0.7706	0.05645	1.892	—	—	—
2024 volatility	0.7706	0.05600	1.883	—	—	—
2025 volatility	0.7705	0.05606	1.873	—	—	—
12-month FEVD	—	—	—	54.2%	18.7%	27.1%
24-month FEVD	—	—	—	48.9%	21.3%	29.8%
36-month FEVD	—	—	—	47.2%	22.1%	30.7%

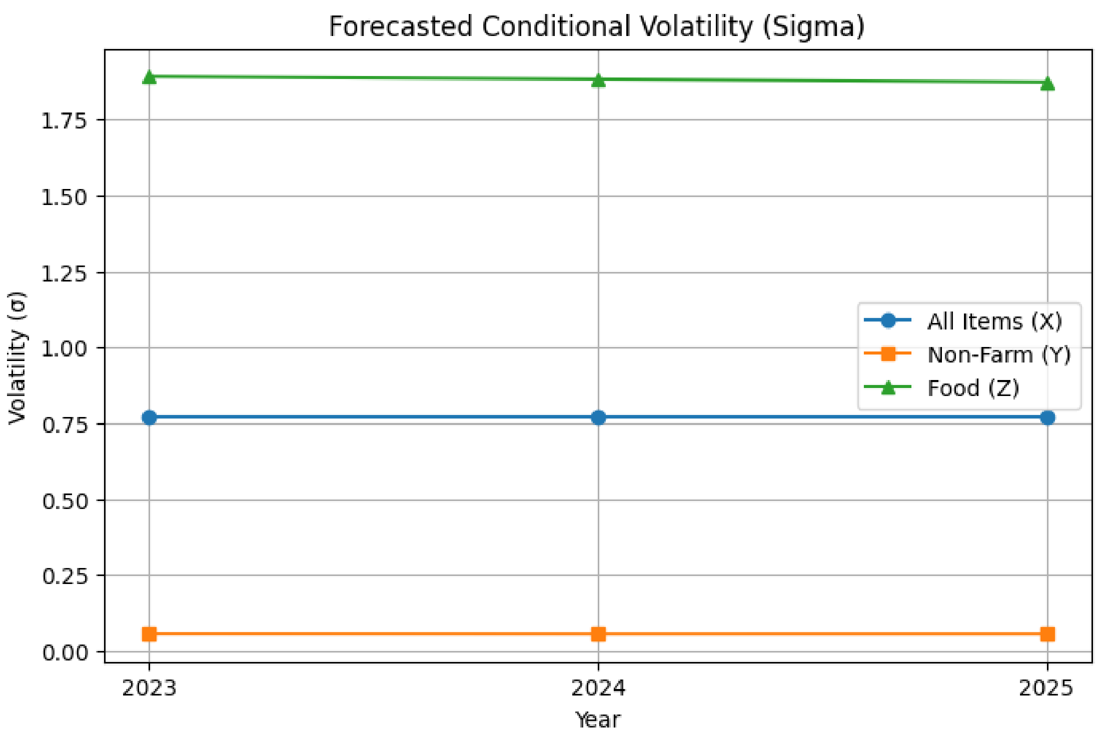

Figure 2. Forecasted conditional volatility for the three CPI components, 2023–2025.

Food-price volatility was forecast to remain near 1.9, approximately 2.5 times the All Items volatility and more than 33 times the non-farm volatility. Its modest decline by 2025 does not alter its dominant risk contribution. FEVD results show that All Items innovations explain all one-month forecast error, but the share falls to 47.2% at 36 months as cross-component transmission grows. Food shocks explain 30.7% at the longest horizon, compared with 22.1% for non-farm shocks.

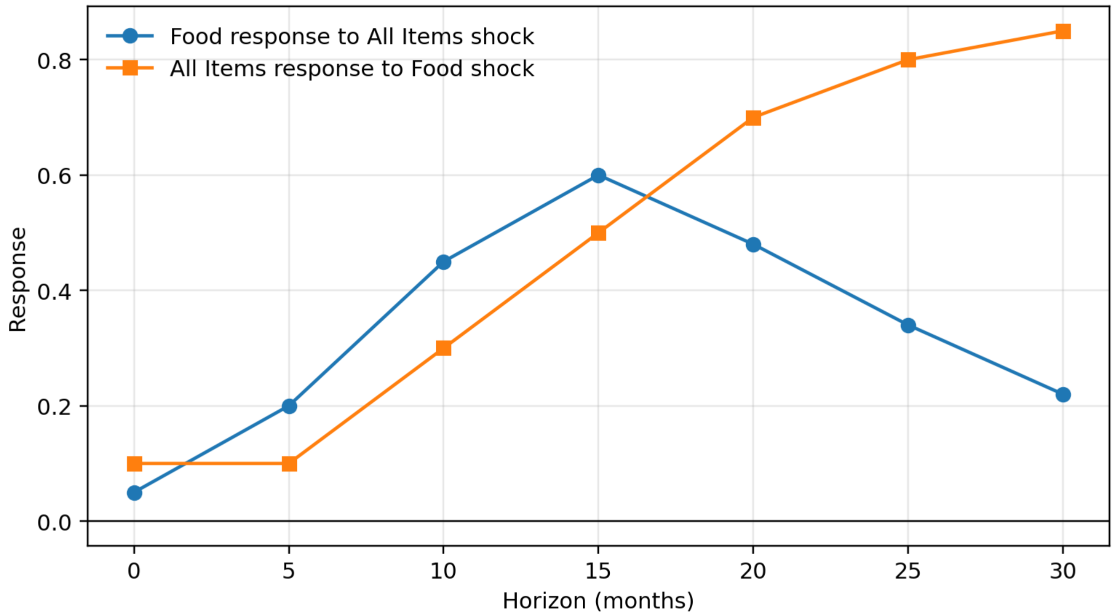

Figure 3. Comparative impulse responses between Food CPI and All Items CPI.

IRFs reinforce this interpretation. A shock to All Items produced a positive food-price response that peaked around 0.60 and decayed slowly. In the reverse direction, a food-price shock generated an All Items response that reached about 0.85 and remained positive throughout the horizon. The larger and more persistent response of overall CPI to food shocks identifies food-price stabilisation as a high-leverage policy target.

3.6. Discussion

The results demonstrate why a single mean or variance model is insufficient for Nigerian CPI. The VAR captured interdependence and removed residual serial correlation, but non-normality and ARCH effects remained. The fGARCH stage therefore addressed information that was systematically omitted from the conditional mean. This complementarity explains the strong forecast gains of the hybrid model and is consistent with the broader GARCH literature, which treats volatility clustering as a separate dynamic process rather than random noise (Engle, 1982; Bollerslev, 1986).

High $\alpha + \beta$ values indicate that inflation uncertainty does not disappear rapidly after a disturbance. The result is particularly important for policy communication: agents may revise expectations for an extended period following currency, supply or policy shocks. The long-memory evidence also suggests that future work should compare the present functional specification with explicitly fractionally integrated volatility models. The negative EGARCH coefficient for food CPI implies asymmetric responses, consistent with market uncertainty about whether downward price movements are temporary or durable. Such asymmetry is relevant because policy announcements that are not credible or well communicated may increase, rather than reduce, uncertainty.

Food-price dominance is supported by three separate results: the highest Hurst exponent, the largest immediate ARCH response, and the largest external contribution to long-horizon All Items forecast uncertainty. This triangulation strengthens the substantive conclusion. Food inflation in Nigeria is shaped by agricultural productivity, transport and storage costs, exchange rates, insecurity, trade restrictions and global commodity markets. Monetary tightening alone may therefore be insufficient where supply-side constraints are the primary drivers. A coordinated approach combining credible monetary policy with food-system interventions is more consistent with the observed transmission mechanism.

Some coefficient signs, especially the negative effects involving non-farm CPI, are counterintuitive and should not be interpreted causally. They may reflect substitution, the construction of overlapping CPI indices, multicollinearity among components or omitted macroeconomic variables. The analysis identifies predictive relationships rather than structural causal effects. This distinction is important for proceeding readers and motivates structural VAR, exogenous-variable and disaggregated-sector extensions.

4. Conclusion

This study developed a hybrid VAR–fGARCH framework for the joint modelling of Nigerian CPI mean and volatility dynamics. All three price indices were I(1), exhibited multiple structural breaks and displayed strong long memory. VAR(2) adequately modelled the conditional mean, but residual non-normality and ARCH effects required a conditional-variance stage. Volatility shocks were highly persistent, and food CPI showed the strongest sensitivity to new information and the most pronounced asymmetric response. The hybrid model significantly outperformed standalone alternatives, while IRF and FEVD results established food shocks as a major source of overall CPI uncertainty.

The main policy implication is that food-price stabilisation should be treated as a core component of inflation management. Strategic reserves, improved storage and logistics, reliable market information, targeted support for productive supply chains and reduced internal trade frictions can moderate food-price shocks. Monetary authorities should complement these measures with clear and credible communication to reduce asymmetric uncertainty. Social-protection systems should be strengthened because persistent food volatility disproportionately affects vulnerable households. Investors and businesses should incorporate sector-specific volatility and persistent inflation risk into budgeting, pricing and portfolio decisions.

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