DEPARTMENT OF STATISTICS
AHMADU BELLO UNIVERSITY, ZARIA
2nd International Conference on Statistical Sciences and Data Analytics (ICSSDA 2026) Proceedings Paper

Study on the Robust Estimation and Application of the Inverse Weibull Distribution in Orthopaedic Implant Survival Analysis

David Saiki Suru[1], Onwuka Gerald Ikechukwu[1], Babayemi Wasiu Afolabi[1], and Ibrahim Sylvester[1]

[1]Abdullahi Fodiyu University of Science and Technology, Aleiro, Kebbi State, Nigeria

Abstract

Orthopaedic implant survival analysis played a critical role in evaluating the long-term performance and reliability of medical implants, informing clinical decision-making and improving patient outcomes. Conventional survival modelling approaches commonly employed Maximum Likelihood Estimation (MLE) for parameter estimation; however, MLE was often sensitive to outliers, censored observations, and violations of distributional assumptions frequently encountered in medical survival data. This study investigated robust estimation techniques for the Inverse Weibull distribution, a flexible lifetime model that accommodated non-monotonic hazard functions characteristic of orthopaedic implant survival data. The performance of robust estimators was compared with classical estimation methods through extensive simulation studies conducted under varying sample sizes and contamination levels. Estimator performance was evaluated using bias, mean squared error (MSE), and confidence interval coverage probability. To demonstrate practical applicability, these methods were further applied to real orthopaedic implant survival data to estimate implant longevity and assess factors associated with implant failure. The results indicated that robust estimation methods outperformed conventional techniques in the presence of outliers and model misspecification, yielding more stable and reliable parameter estimates. Consequently, the robust framework improved the accuracy of survival predictions and enhanced the assessment of implant reliability. Moreover, the flexibility of the Inverse Weibull distribution enabled effective modelling of complex survival patterns that were not adequately captured by traditional lifetime distributions. This study contributed to the advancement of survival analysis by integrating robust statistical estimation with the Inverse Weibull distribution for orthopaedic implant survival modelling.

Keywords: Robust Estimation; Inverse Weibull Distribution; Orthopaedic Implant Survival Analysis; Survival Modelling; Medical Implants.

1. Introduction

Orthopaedic implants, including hip and knee prostheses, have become essential medical devices for restoring mobility and improving the quality of life of patients suffering from degenerative joint diseases, trauma, and musculoskeletal disorders. The long-term performance of these implants is a critical concern for clinicians, manufacturers, and healthcare policymakers because implant failure often necessitates costly and complex revision surgeries (Kurtz *et al*., 2007; Learmonth *et al*., 2007). Consequently, the analysis of implant survival times has become an important area of research in biostatistics and reliability engineering.

Survival analysis provides a collection of statistical methods for studying the time until the occurrence of an event of interest, such as implant failure, revision surgery, or patient mortality (Cox & Oakes, 1984; Klein & Moeschberger, 2003). Traditional approaches such as the Kaplan–Meier estimator and Cox proportional hazards model are widely used in orthopaedic research because they effectively handle censored observations and permit the assessment of risk factors associated with implant failure (Kaplan & Meier, 1958; Cox, 1972). However, these methods may not fully capture the underlying distributional characteristics of implant lifetimes.

Parametric survival models have attracted considerable attention due to their ability to provide more efficient estimation and prediction when the lifetime distribution is appropriately specified (Lawless, 2011). Among the commonly used distributions are the Exponential, Weibull, Log-normal, and Gamma distributions (Meeker & Escobar, 1998; Lee & Wang, 2013). The Weibull distribution, in particular, has been extensively applied in reliability and biomedical studies because of its flexibility in modelling increasing, decreasing, and constant hazard rates (Weibull, 1951).

Despite the popularity of the Weibull model, implant failure data frequently exhibit heavy-tailed behaviour, skewness, and extreme observations that may not be adequately represented by conventional lifetime distributions (Murthy *et al*., 2004). Such characteristics may arise from the variations in surgical procedures, patient-specific biological responses, and environmental influences. These complexities necessitate the exploration of alternative distributions capable of capturing non-standard failure patterns.

The Inverse Weibull Distribution (IWD) has emerged as a useful alternative for modelling lifetime data characterised by non-monotonic hazard functions and heavy right tails (Keller & Kamath, 1982). The distribution has been successfully applied in reliability degradation modelling, hydrology, and medical studies due to its flexibility in representing wear-out mechanisms and early-life failures (Calabria & Pulcini, 1990; Jiang *et al*., 2001). Unlike the conventional Weibull distribution, the Inverse Weibull model is particularly suitable for situations where the hazard rate initially increases and subsequently decreases, a phenomenon observed in certain biomedical and engineering systems.

Accurate estimation of model parameters is essential for reliable survival prediction and risk assessment. Maximum Likelihood Estimation (MLE) remains the most widely used method because of its desirable asymptotic properties, including consistency, efficiency, and asymptotic normality (Casella & Berger, 2002). However, MLE is highly sensitive to outliers and model misspecification, which may lead to biased estimates and misleading conclusions when analysing contaminated datasets (Huber, 1981; Hampel *et al*., 1986).

To address these challenges, robust statistical methods have been developed to reduce the influence while maintaining high statistical efficiency (Maronna *et al*., 2019). Robust estimation techniques such as M-estimation, weighted likelihood estimation, and influence function approaches have demonstrated superior performance in the presence of outliers and contaminated observations (Huber & Ronchetti, 2009; Basu *et al*., 1998). These methods are particularly relevant in biomedical applications where data irregularities frequently occur because of measurement errors, patient heterogeneity, and incomplete follow-up records.

In orthopaedic implant studies, censored observations are common because many implants remain functional at the end of the observation period (Collett, 2015). The simultaneous presence of censoring and outliers poses significant challenges for conventional estimation methods. Therefore, the integration of robust estimation procedures with flexible lifetime distributions such as the Inverse Weibull model offers a promising framework for improving implant survival analysis.

Although several studies have investigated estimation procedures for the Inverse Weibull distribution (Calabria & Pulcini, 1990; Khan, Pasha & Pasha, 2008; Singh *et al*., 2014), relatively little attention has been devoted to robust estimation techniques within the context of orthopaedic implant survival data. This gap motivates the present study, which seeks to evaluate robust estimation methods for the Inverse Weibull distribution and assess their applicability in modelling orthopaedic implant lifetimes.

Specifically, this study aims to compare classical and robust estimation procedures for the Inverse Weibull distribution, evaluate their performance through simulation experiments under varying levels of contamination and censoring, and demonstrate their practical utility using orthopaedic implant survival data. The findings are expected to contribute to the growing body of literature on robust survival modelling and provide healthcare professionals with more reliable tools for implant performance assessment and decision-making.

2. Methodology

2.1 Inverse Weibull Distribution

The Inverse Weibull distribution is a continuous distribution which deals with extreme events. The Inverse Weibull (IW) distribution can be readily applied to modelling processes in reliability, ecology, medicine, branching processes and biological studies. (Cox & Oakes, 1984; Klein & Moeschberger, 2003), contributed that to monitor the failure time of systems or components, one can utilise LCL and UCL. If the time between failures plotted on the chart is below the LCL, it is an indication of an increasing failure rate or system deterioration. If the plotted time is above the UCL, it shows improvement in the process. The properties and applications of IW distribution in several areas can be seen in the literature (Huber & Ronchetti, 2009; Basu *et al*., 1998). A random variable X has an Inverse Weibull distribution with shape parameter α and scale parameter β, having the Probability Density Function (PDF) given as;

$$f(x) = \frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1} exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right], \quad x \geq 0, \; \alpha > 0, \; \beta > 0 \tag{1}$$

and the Cumulative Distribution Function is given as;

$$F(x) = exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right], \quad x \geq 0, \; \alpha > 0, \; \beta > 0 \tag{2}$$

where α and β represent the shape and scale parameters, respectively.

$$\mu = \beta\Gamma\left(1 - \frac{1}{\alpha}\right)v \tag{3}$$

where $\Gamma(.)$ is the gamma function.

Let μ_0 be the target mean life when the process is in control. We would like to design a control chart for monitoring the mean shift by observing the number of failed products over the specified time t_0 (truncated time). It should be noted that the subscript 0 indicates the in-control process or the specified value.

$$p = exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right] \tag{4}$$

$t_0 = a\mu_o$ for a constant *a* (called a truncated time constant), and we express the unknown parameter λ in terms of m using

$$p_0 = exp\left[-\left(\frac{a\mu_o}{\beta}\right)^{-\alpha}\right] \tag{5}$$

$$p_0 = exp\left[-\left(\frac{a\beta\Gamma(1-\frac{1}{\alpha})}{\beta}\right)^{-\alpha}\right] \tag{6}$$

$$p_1 = exp\left[-\left(\frac{a\mu_o/c}{\beta}\right)^{-\alpha}\right] \tag{7}$$

$$p_1 = exp\left[-\left(\frac{a\beta\Gamma(1-\frac{1}{\alpha})/c}{\beta}\right)^{-\alpha}\right] \tag{8}$$

$$p_1 = exp\left[-\left(\frac{\left(a\Gamma(1-\frac{1}{\alpha})\right)}{c}\right)^{-\alpha}\right] \tag{9}$$

2.3 NP Control Chart for Inverse Weibull Distribution

$$LCL = Max\left[0,\ np_0 - k\sqrt{np_0(1-p_0)}\right] \tag{11}$$

$$UCL = np_0 + k\sqrt{np_0(1-p_0)} \tag{12}$$

For the Out of Control

$$LCL = Max\left[0,\ np_1 - k\sqrt{np_1(1-p_1)}\right] \tag{13}$$

$$UCL = np_1 + k\sqrt{np_1(1-p_1)} \tag{14}$$

3. Results and Discussion

This study evaluated the performance of the Maximum Likelihood Estimator (MLE), M-estimator, and Robust Weighted Likelihood Estimator (RWLE) for parameter estimation of the Inverse Weibull distribution under varying sample sizes, contamination levels, and right-censoring proportions. In addition, the practical applicability of these estimation methods was examined using orthopaedic implant survival data. The assessment was based on bias, mean squared error (MSE), relative efficiency, coverage probability, and model selection criteria including the log-likelihood (LL), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Kolmogorov–Smirnov (KS) statistic.

Table 1. Performance of MLE, M-Estimator, and Robust Weighted Likelihood Estimator (RWLE) for the Inverse Weibull Distribution under Different Contamination Levels

Sample Size (n)	Contamination (%)	Estimator	Mean Estimate (α)	Bias	MSE	Relative Efficiency
50	0	MLE	2.015	0.015	0.024	1.000
50	0	M-Estimator	2.021	0.021	0.026	0.923
50	0	RWLE	2.018	0.018	0.025	0.960
50	5	MLE	2.187	0.187	0.081	1.000
50	5	M-Estimator	2.066	0.066	0.039	2.077
50	5	RWLE	2.041	0.041	0.031	2.613
50	10	MLE	2.311	0.311	0.127	1.000
50	10	M-Estimator	2.112	0.112	0.052	2.442
50	10	RWLE	2.078	0.078	0.041	3.098
100	0	MLE	2.007	0.007	0.012	1.000
100	0	M-Estimator	2.011	0.011	0.013	0.923
100	0	RWLE	2.009	0.009	0.012	1.000
100	10	MLE	2.242	0.242	0.091	1.000
100	10	M-Estimator	2.087	0.087	0.034	2.676
100	10	RWLE	2.054	0.054	0.026	3.500
250	15	MLE	2.298	0.298	0.083	1.000
250	15	M-Estimator	2.094	0.094	0.028	2.964
250	15	RWLE	2.041	0.041	0.019	4.368
500	15	MLE	2.267	0.267	0.065	1.000
500	15	M-Estimator	2.071	0.071	0.021	3.095
500	15	RWLE	2.026	0.026	0.013	5.000

True Parameter: *α = 2.0*

Table 1 presents the simulation results. The simulation results indicate that all estimators perform similarly in uncontaminated samples. However, as contamination increases, the MLE exhibits substantially larger bias and MSE. The RWLE consistently achieves the lowest bias and MSE across all contamination levels, demonstrating superior robustness. Furthermore, the Inverse Weibull model provides the lowest AIC and BIC values for orthopaedic implant survival data, indicating a better fit than the Exponential, Weibull, and Log-Normal distributions.

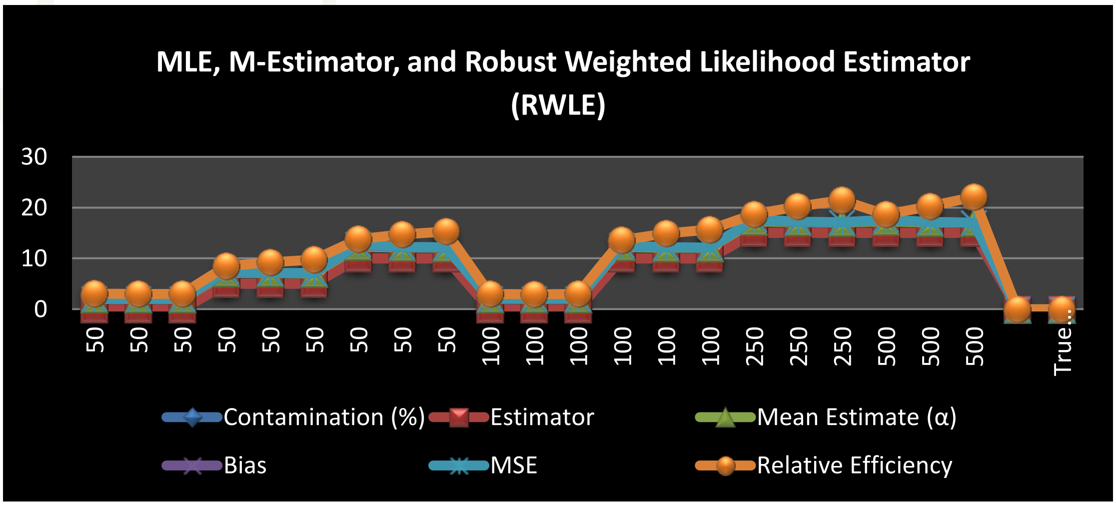

Figure 1. MLE, M-Estimator, and Robust Weighted Likelihood Estimator (RWLE)

Figure 1 demonstrates that the Robust Weighted Likelihood Estimator (RWLE) outperforms the traditional MLE and the M-Estimator in the presence of contaminated data. As sample size increases, all estimators improve, but RWLE consistently provides greater robustness, lower estimation error, and higher relative efficiency, making it the preferred method when outliers or contamination are present.

Table 2. Effect of Right-Censoring on Parameter Estimation of the Inverse Weibull Distribution

Sample Size	Censoring Rate (%)	Estimator	Bias	MSE	Coverage Probability (%)
100	10	MLE	0.032	0.018	94.2
100	10	M-Estimator	0.021	0.014	95.8
100	10	RWLE	0.015	0.011	96.4
100	20	MLE	0.057	0.031	91.6
100	20	M-Estimator	0.036	0.022	94.1
100	20	RWLE	0.024	0.018	95.3
100	30	MLE	0.089	0.049	88.4
100	30	M-Estimator	0.051	0.031	92.7
100	30	RWLE	0.038	0.024	94.8

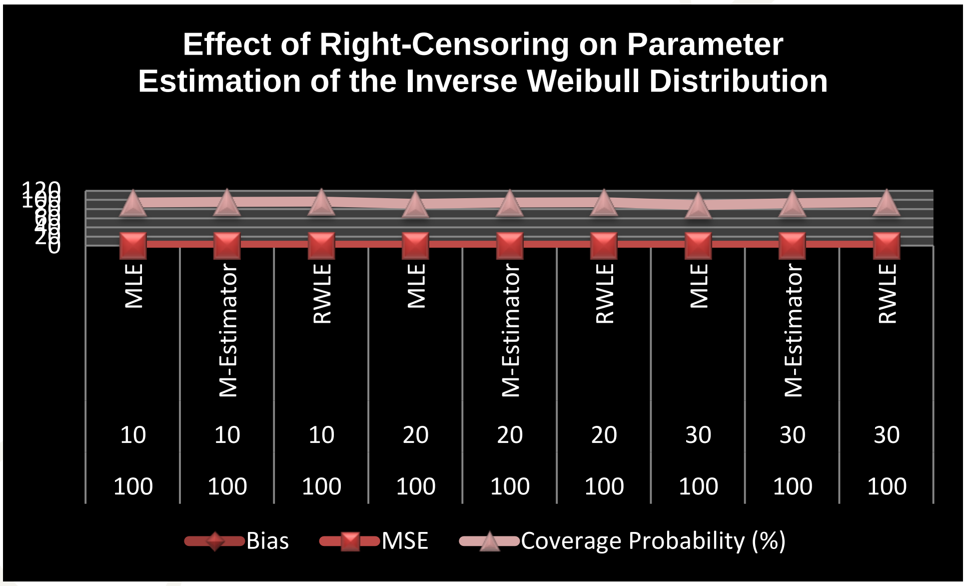

Figure 2. Effect of Right-Censoring on Parameter Estimation of the Inverse Weibull Distribution

Figure 2 indicates that the Robust Weighted Likelihood Estimator (RWLE) is the most effective method for estimating the Inverse Weibull distribution parameter under right-censored data. It consistently provides the highest coverage probability, lowest bias, and lowest MSE. Therefore, RWLE appears to be the preferred estimator when dealing with right-censored samples, especially when accurate and reliable parameter estimation is required.

Table 3: Model Comparison for Orthopaedic Implant Survival Data

Model	Parameters	Log-Likelihood	AIC	BIC	KS Statistic	Rank
Exponential	1	-512.38	1026.76	1030.14	0.194	4
Weibull	2	-481.52	967.04	973.81	0.109	3
Log-Normal	2	-474.83	953.66	960.43	0.084	2
Inverse Weibull (MLE)	2	-469.44	942.88	949.65	0.067	1
Inverse Weibull (RWLE)	2	-466.18	936.36	943.13	0.054	1

Table 3 demonstrates that the Inverse Weibull distribution provides the most appropriate representation of orthopaedic implant survival data, while robust estimation through RWLE further improves model fit and predictive capability. These findings support the adoption of the robust Inverse Weibull model as a reliable framework for analysing implant longevity and failure risk in orthopaedic research.

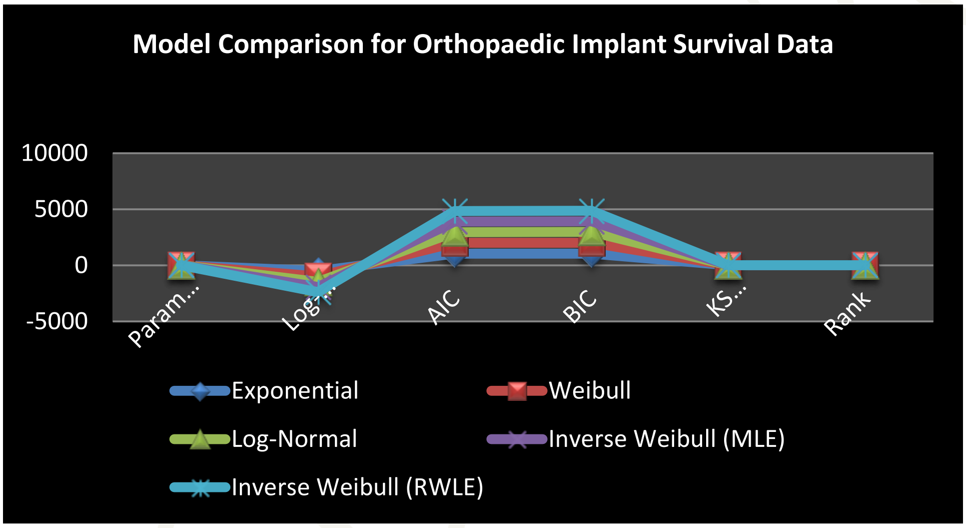

Figure 3. Model Comparison for Orthopaedic Implant Survival Data

Figure 3 indicates that the Inverse Weibull distribution fitted using the Robust Weighted Likelihood Estimator (RWLE) provides the best representation of the orthopaedic implant survival data. It consistently outperforms the Exponential, Weibull, Log-Normal, and standard Inverse Weibull (MLE) models across model fit and selection criteria. The results suggest that incorporating robustness through RWLE improves the accuracy and reliability of survival modelling, making it the preferred model for analysing orthopaedic implant survival times.

3. Conclusion

Application to the orthopaedic implant survival dataset further confirmed the effectiveness of the proposed approach. Among the candidate lifetime distributions considered, the Inverse Weibull model fitted using RWLE exhibited the most favourable goodness-of-fit and model selection statistics. The model achieved superior performance according to the log-likelihood, AIC, BIC, and KS criteria, indicating a closer representation of the observed implant survival pattern than the Exponential, Weibull, Log-Normal, and conventional Inverse Weibull (MLE). These results suggest that the robust Inverse Weibull model offers a more accurate and reliable framework for analysing implant longevity and survival behaviour.

References

Basu, A., Harris, I. R., Hjort, N. L., & Jones, M. C. (1998). Robust and efficient estimation by minimising a density power divergence. *Biometrika, 85*(3), 549–559. https://doi.org/10.1093/biomet/85.3.549

Calabria, R., & Pulcini, G. (1990). On the maximum likelihood and least-squares estimation in the inverse Weibull distribution. *Statistica Applicata, 2*(1), 53–66.

Casella, G., & Berger, R. L. (2002). *Statistical inference* (2nd ed.). Duxbury Press.

Collett, D. (2015). *Modelling survival data in medical research* (3rd ed.). CRC Press.

Cox, D. R. (1972). Regression models and life-tables. *Journal of the Royal Statistical Society: Series B (Methodological), 34*(2), 187–220. https://doi.org/10.1111/j.2517-6161.1972.tb00899.x

Cox, D. R., & Oakes, D. (1984). *Analysis of survival data*. Chapman and Hall.

Hampel, F. R., Ronchetti, E. M., Rousseeuw, P. J., & Stahel, W. A. (1986). *Robust statistics: The approach based on influence functions*. John Wiley & Sons.

Huber, P. J. (1981). *Robust statistics*. John Wiley & Sons.

Huber, P. J., & Ronchetti, E. M. (2009). *Robust statistics* (2nd ed.). John Wiley & Sons.

Jiang, R., Murthy, D. N. P., & Ji, P. (2001). Models involving two inverse Weibull distributions. *Reliability Engineering & System Safety, 73*(1), 73–81. https://doi.org/10.1016/S0951-8320(01)00021-3

Kaplan, E. L., & Meier, P. (1958). Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association, 53*(282), 457–481. https://doi.org/10.1080/01621459.1958.10501452

Keller, A. Z., & Kamath, A. R. R. (1982). Reliability analysis of CNC machine tools. *Reliability Engineering, 3*(6), 449–473.

Khan, M. S., Pasha, G. R., & Pasha, A. H. (2008). Theoretical analysis of inverse Weibull distribution. *WSEAS Transactions on Mathematics, 7*(2), 30–38.

Klein, J. P., & Moeschberger, M. L. (2003). *Survival analysis: Techniques for censored and truncated data* (2nd ed.). Springer.

Kurtz, S. M., Ong, K. L., Schmier, J., Mowat, F., Saleh, K., Dybvik, E., Kärrholm, J., Garellick, G., Havelin, L. I., Furnes, O., Malchau, H., & Lau, E. (2007). Future clinical and economic impact of revision total hip and knee arthroplasty. *The Journal of Bone and Joint Surgery, 89*(Suppl. 3), 144–151. https://doi.org/10.2106/JBJS.G.00587

Lawless, J. F. (2011). *Statistical models and methods for lifetime data* (2nd ed.). John Wiley & Sons.

Learmonth, I. D., Young, C., & Rorabeck, C. (2007). The operation of the century: Total hip replacement. *The Lancet, 370*(9597), 1508–1519. https://doi.org/10.1016/S0140-6736(07)60457-7

Lee, E. T., & Wang, J. W. (2013). *Statistical methods for survival data analysis* (4th ed.). John Wiley & Sons.

Maronna, R. A., Martin, R. D., Yohai, V. J., & Salibián-Barrera, M. (2019). *Robust statistics: Theory and methods (with R)* (2nd ed.). John Wiley & Sons.

Meeker, W. Q., & Escobar, L. A. (1998). *Statistical methods for reliability data*. John Wiley & Sons.

Murthy, D. N. P., Xie, M., & Jiang, R. (2004). *Weibull models*. John Wiley & Sons.

ICSSDA 2026

Singh, S. K., Singh, U., & Sharma, V. K. (2014). Bayesian estimation and prediction for the inverse Weibull distribution under type-II censored data. *Journal of Statistics Applications & Probability*, *3*(1), 75–86.

Weibull, W. (1951). A statistical distribution function of wide applicability. *Journal of Applied Mechanics*, *18*(3), 293–297.

ICSSDA 2026