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Fréchet-Based Hankel Integral Transform for Extreme and Oscillatory Pharmacokinetics: A Systematic Literature Review

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Abstract

Hankel integral transforms are important mathematical tools for analysing oscillatory phenomena, radial systems, and Bessel-function-based processes. Despite extensive theoretical advances, their integration with heavy-tailed probability models remains limited. This PRISMA-guided systematic literature review examined theoretical developments, applications, distributional integrations, and research opportunities using PubMed, Scopus, Web of Science, Science Direct, Google Scholar, and other sources. From 1,388 retrieved records, rigorous screening, quality assessment, and content evaluation identified 15 studies and 2 reports for synthesis. Findings showed significant advances in classical, fractional, multidimensional, and generalised Hankel transform methods. However, most studies remained theoretical, with few practical applications. Heavy-tailed probability distributions were rarely incorporated, while pharmacokinetic and biomedical applications were largely absent. Oscillatory modelling and extreme-value analysis also evolved largely as separate research domains. The review identifies a critical gap in integrating Hankel transforms with Fréchet-type distributions and highlights the absence of a unified framework for modelling oscillatory behaviour under extreme variability. These findings provide a strong theoretical basis for developing Fréchet-based Hankel transform frameworks for heavy-tailed, oscillatory, and pharmacokinetic systems.

Keywords: Hankel transform; Integral transforms; Fréchet distribution; Pharmacokinetic modelling; Oscillatory processes; Systematic literature review.

1. Introduction

The analysis of oscillatory and highly variable phenomena remains a central challenge across mathematical physics, engineering, and biomedical sciences. Integral transforms have historically provided powerful analytical frameworks for simplifying complex systems by converting differential problems into more tractable forms. Among these transforms, the Hankel integral transform occupies a unique position due to its natural association with radial symmetry and Bessel-function

kernels, making it particularly suitable for modelling wave propagation, diffusion processes, imaging systems, and other oscillatory phenomena (Baddour, 2014; Piessens, 2000). Over the past two decades, significant advances have been made in extending the classical Hankel transform through fractional formulations, multidimensional representations, distributional frameworks, convolution operators, and numerical real-order implementations, thereby broadening its theoretical foundations and practical applicability (Betancor, 2005; Waphare & Shaikh, 2023).

At the same time, modern pharmacokinetic (PK) studies increasingly encounter concentration-time profiles characterised by strong oscillations, substantial between-subject heterogeneity, skewness, and heavy-tailed behaviour. These characteristics frequently violate the assumptions underlying traditional Gaussian-based analytical frameworks, resulting in inadequate representation of extreme observations and complex biological variability. Extreme-value distributions, particularly the Fréchet distribution, provide a theoretically sound mechanism for modelling such heavy-tailed behaviour and rare but influential events. Despite the growing relevance of both Hankel-transform theory and Fréchet-based modelling, the existing literature reveals little integration between these two research domains. Furthermore, most applications of integral transforms in PK modelling continue to rely predominantly on Fourier-based or time-domain approaches, while the potential of Hankel-based frameworks for analysing oscillatory processes under extreme variability remains largely unexplored. This disconnect has created a significant theoretical and methodological gap in the literature.

Therefore, this study conducts a systematic literature review (SLR) to critically examine existing developments in Hankel integral transforms, related oscillatory-transform frameworks, and distributional extensions. Guided by the PRISMA methodology, the review synthesises current knowledge, identifies methodological limitations, and establishes a rigorous conceptual foundation for the future development of Fréchet-based Hankel transform models for oscillatory and heavy-tailed pharmacokinetic systems.

2. Materials and Methods

This study adopted the PRISMA framework recommended by Haddaway *et al*. (2022). The review process comprised four phases: formulation of research questions aligned with the study objectives, development of a search design including search terms and selection strategies, data extraction and quality assessment of identified studies, and data synthesis involving final study inclusion and evidence integration (Azihaiwe-Justin *et al*., 2025; Ani *et al*., 2026; Ani *et al*., 2026).

2.1. Research Questions

This SLR synthesises evidence on Hankel transforms, their possible distributional integration, and research gaps. Four research questions (RQs) were formulated to guide the review as follows:

(i.) What are the existing Hankel integral transforms?
(ii.) What are other existing integral transforms used for oscillatory series characterisations?
(iii.) What are the existing Hankel mixtures?
(iv.) What are the limitations of existing Hankel transforms and mixtures?

2.2. Database Search Strategy

The search design involved the search terms, resources (i.e. literature), and search procedure. Finally, identified literature was then retrieved and imported into the Rayyan software for additional screening **(**Ouzzani *et al*., 2016**).**

2.2.1. Literature Search Process

The Boolean search string presented in Table 1 was systematically developed from the formulated RQs by identifying key concepts, relevant synonyms, alternative spellings, and keywords from related literature. Boolean operators (i.e. OR) and truncation techniques were employed to enhance search sensitivity, comprehensiveness, and retrieval efficiency consistent with established SLR guidelines (Ani *et al*., 2026; Ani *et al*., 2026; Haddaway *et al*., 2022).

Table 1. Keywords: Mixtures for Literature Retrieval

(Hankel* OR "Hankel transform" OR "Hankel-Fréchet" OR "Hankel Frechet" OR oscillatory* OR Quasi-oscillatory* OR fluctuat* OR periodic* OR cyclic* OR radia*) OR ("pharmacokinetics" OR pharmacokinetic* OR "population PK" OR "Population-level PK" OR "pharmacodynamics" OR pharmacodynamics* OR "population PD" OR "Population-level PD") OR ("heavy-tail*" OR "extreme value distribution*" OR "distributional theor*" OR elliptical distribution*) OR ("Bessel function" OR "Bessel equations" OR Bessel equation* OR Bessel differential*) OR ("epidemiolog*" OR "disease" OR "public-health" OR "global health" OR "health") OR ("time-series*" OR "time series*" OR series*)

2.2.2. Indexed Database Records

This study utilised secondary data obtained from literature identified through the search query in Table 1. Relevant publications were retrieved from PubMed, Web of Science, Google Scholar, Science Direct, Scopus, and other sources. Searches were conducted primarily using titles and abstracts to identify peer-reviewed journal articles, book chapters, conference proceedings, workshops, and symposium publications.

2.2.3.Study Identification Procedure

As previously noted, this SLR employed a comprehensive literature search strategy (see Fig. 1). An exhaustive search was conducted across the five electronic databases listed in Section 2.2.2, and all retrieved studies were compiled as potential resources. Additionally, reference lists of identified studies were manually examined in line with the RQs to locate further relevant records for inclusion.

2.3. Study Screening and Selection

Study selection followed the PRISMA framework through identification, screening, eligibility, and inclusion. Searches across five databases yielded 1,388 records; 612 were removed before screening. Of 776 screened using eligibility, quality, and content criteria (Tables 2-4), 748 were excluded. Twenty-eight full texts were assessed, with further exclusions applied. Five additional reports were identified, leaving 15 studies and 2 reports for final synthesis. As illustrated in Fig. 1, all eligible studies were systematically managed using Mendeley to facilitate structured screening, transparent tracking, and consistent citation management throughout the review process.

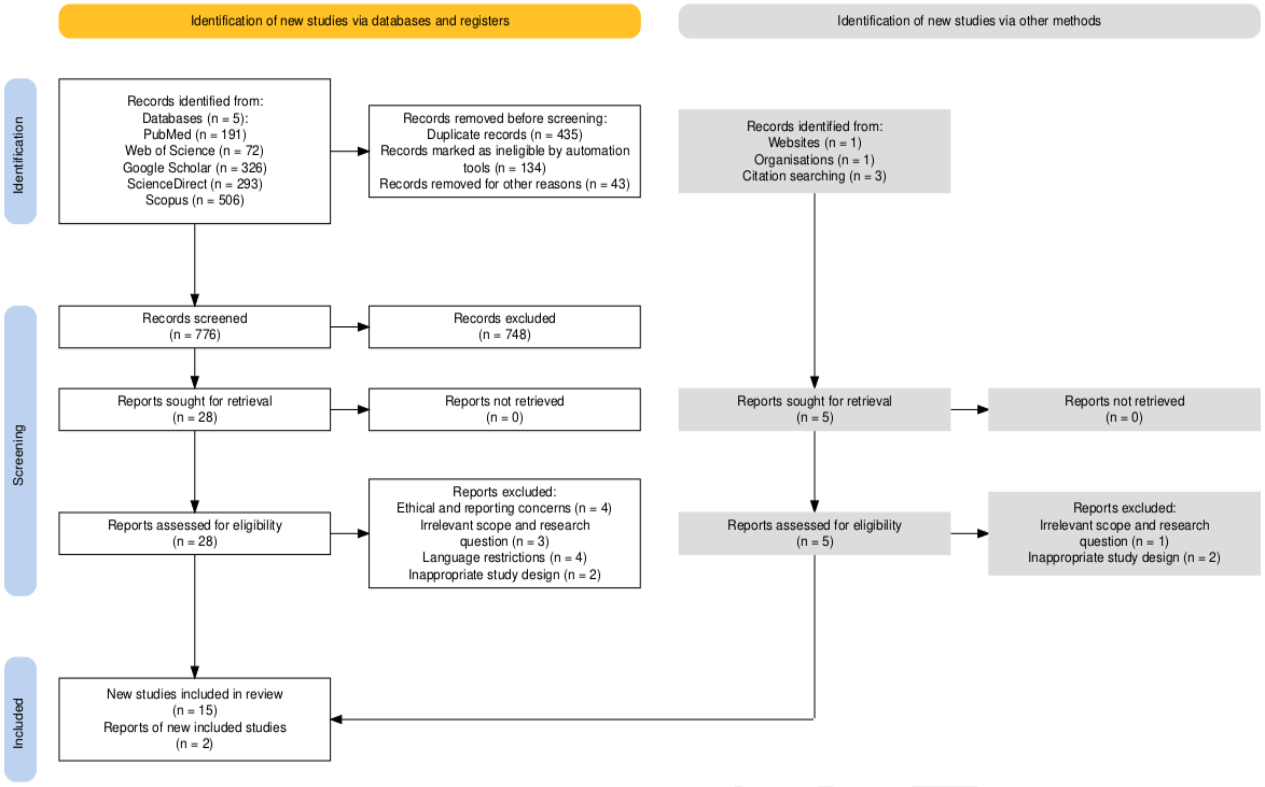

Figure 1. PRISMA-based study selection flowchart

2.3.1. Inspection

As shown in Figure 1, 1,388 studies were identified and systematically screened against the study aim, research questions, and assessment criteria. Titles and full texts were reviewed, excluding studies not addressing the study aim. Only English-language, peer-reviewed publications and scholarly sources were included in Rayyan. Duplicate records were removed, retaining the most complete versions. The SLR covered Hankel and related integral transforms published between January 1, 2000 and December 31, 2025 (see Table 2).

Table 2. Study Selection Criteria

S/N	Inclusion Criteria	Exclusion Criteria
i	Studies published between January 1, 2000 and December 31, 2025.	Grey literature or publications with incomplete bibliographic information.
ii	English-language (or officially translated) peer-reviewed scholarly publications, including journals, conference papers, books, and book chapters.	Non-English publications without accessible English versions and non-scholarly sources (e.g., editorials, commentaries, newsletters).
iii	Studies on Hankel transforms, related integral transforms, oscillatory phenomena, Bessel frameworks, extreme-value distributions, or pharmacokinetic modelling.	Studies outside the review scope or not addressing Hankel transforms, related mathematical frameworks, or the research questions.
iv	Empirical, theoretical, computational, simulation, methodological, or applied studies addressing at least one research question.	Studies with insufficient methodological details, incomplete results, or inaccessible full texts.
v	Studies providing adequate methodological, theoretical, numerical, or application details for meaningful synthesis.	Studies lacking scholarly validation or relying solely on unverified self-reported information.

Note: Duplicate records identified across databases were removed, retaining only the most complete and recent version for assessment and inclusion.

2.3.2. Critical Appraisal Checklist

The critical appraisal questions (CAQs) of selected literature were attained using a scoring/weighting technique to obtain relevant studies that are capable of providing answers to the stated research questions. Some CAQs were formulated so as to assess the applicability, relevance, completeness, and credibility of the studies selected.

Table 3. Critical Appraisal Questions (CAQs)

S/N	Assessment Questions
i.	Are the aims/objectives of the study plainly stated?
ii.	Is the Hankel transform clearly stated?
iii.	Is the Hankel integral transform applied to experimental or simulated datasets?
iv.	Is the Hankel mixture form or Hankel distribution form clearly stated?
v.	Are the properties and behaviour of the Hankel transform clearly stated?
vi.	Are there any limitations concerning the Hankel or related transform used? Are these limitations clearly stated?

Table footnote: Yes = 1 **s**core if the CAQ is answered; **Partially** = 0.5 score if the CAQ is moderately answered; **No** = 0 score if the CAQ is not answered.

The CAQs in Table 3 were scored as Yes (1), Partially (0.5), or No (0), with total scores determining study quality. Assessment disagreements were resolved by author consensus. Only studies scoring >3 (>50%) were included. Of 776 screened records, 748 were excluded, leaving 28 for detailed assessment. In line with the CAQs, nine more studies were excluded, leaving 15 studies and 2 reports, which met the eligibility and quality criteria (see Table 3.1).

Table 3.1. Critical Appraisal Scores of the Selected Studies

S/No.	Record ID	CAQ1	CAQ2	CAQ3	CAQ4	CAQ5	CAQ6	Total Score
1.	G711	1	0.5	0.5	0.5	0.5	0.5	3.5
2.	G531	1	1	1	1	0.5	1	5.5
3.	G017	1	0.5	0.5	1	0.5	0.5	4
4.	G039	1	1	0.5	0.5	1	0.5	4.5
5.	G232	1	0.5	0.5	0.5	0.5	0.5	3.5
6.	G151	1	0.5	1	1	1	1	5.5
7.	G701	1	1	1	0.5	1	1	5.5
8.	G180	1	1	1	0.5	1	0.5	5
9.	G055	1	1	0.5	1	0.5	1	5
10.	G187	1	0.5	1	1	1	1	5.5
11.	G218	1	1	1	1	1	1	6
12.	G062	1	1	0.5	0.5	1	0.5	4.5
13.	G431	1	1	1	1	1	1	6
14.	G522	1	1	1	1	0.5	0.5	5
15.	G343	1	1	1	1	1	1	6
16.	G691	1	1	1	1	1	1	6
17.	G307	1	1	1	1	1	1	6

Table footnote: G denotes papers with three-digit numbers representing the 748 records initially selected before applying the CAQs; so, after applying the CAQs, 17 studies identified under the records ID column were selected as the most relevant articles to the study aim, having met the quality assessment target score (e.g. G000).

Table 3.1 summarises all eligible studies, including their titles, bibliographic information, objectives, data sources and types, analytical approaches, key findings, and remarks. Guided by the CAQs, these systematically extracted data enhanced review credibility and supported the synthesis of evidence presented in the study.

2.4. Data Synthesis

Data synthesis in this review aims to evaluate and summarise evidence from the selected studies to address the formulated RQs. The 17 eligible studies were critically examined using the CAQs and content evaluation measures (Tables 5 and 4). To enhance precision and coherence, findings were systematically integrated with the review objectives. Both qualitative and quantitative data were synthesised, while evidence relating to RQ1-RQ4 was organised using structured tables and visual presentation tools.

Table 4. Framework for Content Evaluation and Data Extraction

S/N	Evaluation Component	Description
1.	Study Identification and Bibliographic Information	Unique study identifier, publication title, year of publication, authorship details, and source of publication used to facilitate study tracking and referencing.
2.	Publication Category	Classification includes journal articles, conference papers, workshop papers, book chapters, and other review articles.
3.	Research Focus and Objectives	Description of the principal problem addressed, study scope, rationale, and the specific aims or objectives guiding the investigation.
4.	Data Characteristics	Information regarding the origin of the data, data source, nature of the dataset, and classification of the data employed in the study.
5.	Analytical and Methodological Approach	Details of the analytical framework adopted, including the integral transform techniques, hybrid transform models, probability distributions, mathematical formulations, or computational procedures utilised.
6.	Key Outcomes, Findings, and Limitations	Summary of the major findings, conclusions, identified constraints, methodological limitations, and implications arising from the study.

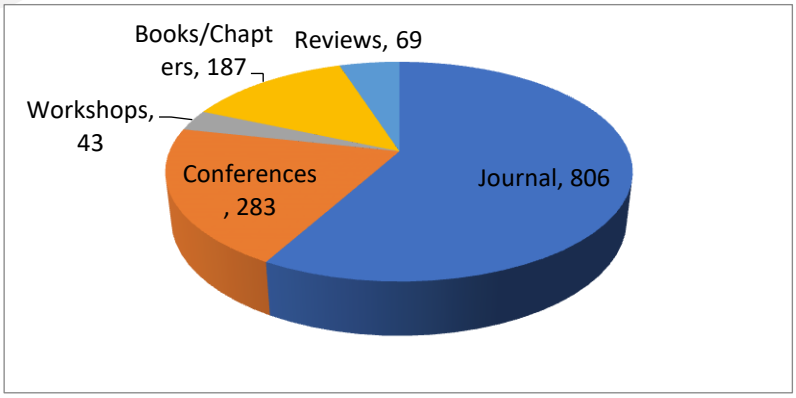

Figure 2. Number of collated resources

Figure 2 provides the number of literature resources according to the type of publication, while Figure 3 shows the number of publications from Jan 1, 2000 to Dec 31, 2025.

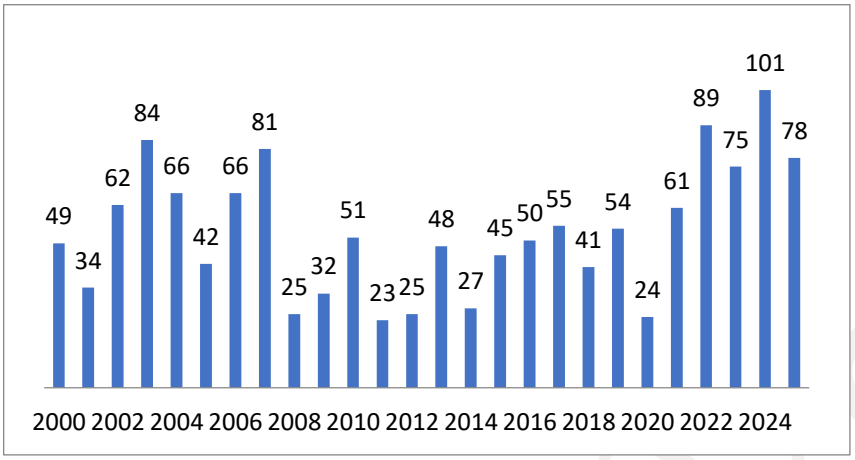

Figure 3. Distribution of publications per year collated from 2000 to 2025

Figure 3 illustrates a generally increasing trend in Hankel-transform-related publications between 2000 and 2025. Publication output accelerated markedly after 2020, reaching its highest levels during 2023-2025, reflecting expanding research activities in fractional, computational, and generalised Hankel-transform formulations.

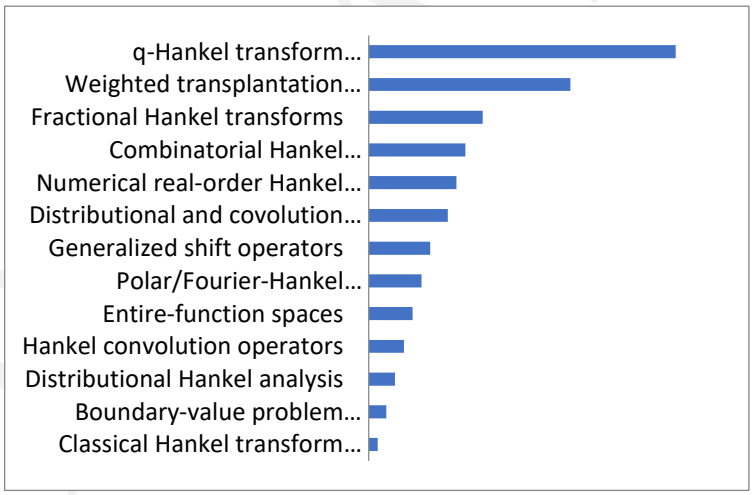

Figure 4. Cumulative emergence of methodological research in Hankel-transform research from 2000 to 2025

Figure 4 shows the cumulative progressive accumulation of distinct methodological areas identified across the selected studies, illustrating the evolution of the field from foundational theory to advanced fractional and computational formulations.

3. Results and Discussions

3.1. An Overview of Selected Studies

This subsection presents an overview of the selected studies (see Table 5);

Table 5. Extracted data from included publications on integral transforms

S/N	Title	Bibliographic details		Study objectives	Methodology		Key findings
		Author's name	Year		Integral transform	Incorporated transforms/ functions	
1.	Abelian and Tauberian results for the fractional Hankel transform of generalised functions.	Atanasova, S., Jakšić, S., Maksimović, S., and Pilipović, S.	2025	The study aims to explore the quasi-asymptotic behaviour of distributions using the fractional Hankel transform (FrHT).	Hankel transform	Fractional Hankel transform	The study showed that the Tauberian result is linked to the asymptotic behaviour of generalised functions in the Zemanian space along with the asymptotics of the fractional Hankel transform.
2.	Uniform weighted inequalities for the Hankel transform transplantation operator	Óscar Ciaurri	2025	To evaluate uniform weighted inequalities for the Hankel transform transplantation operator.	Hankel transform (HT)	None	A weighted vector-valued inequality is obtained. Consequently, the study obtained an extension of a transference theorem due to Rubio de Francia.
3.	Advancements in q-Hankel Transforms Based on Certain Approach of Big q-Bessel Functions and Applications.	Ola A. Ashour, Rajagopalan Ramaswamy and Karima M. Oraby	2025	This study presents a novel variant of a finite q-Hankel transform derived from big q-Bessel Functions.	Hankel transform	Big q-Bessel functions	The study established rigorous conditions under which the finite q-Hankel transforms exhibit only real zeros and demonstrate their adherence to well-defined asymptotic and symmetric patterns.
4.	Fractionalization of Hankel Type Integral Transforms and Their Relevance	B. B. Waphare and R. Z. Shaikh	2023	To extend classical Hankel-type integral transforms by introducing their fractional-order counterparts.	Hankel transform	Barut-Girardello type transforms	The key findings show that fractionalization substantially enlarges the associated class of evolution problems.
5.	A Numerical Algorithm for Arbitrary Real-Order Hankel Transform	Yang Yonglin, Li Xing, Ding Shenghu, and Wang Wenshuai	2022	The study proposed a high-precision and efficient numerical algorithm for calculating the integral of high-oscillation	Hankel transform	None	In the study, the numerical algorithm for the nth-order Hankel transform is studied, and the G-SILTM algorithm is given. The numerical solutions of the examples are in good agreement with the analytical solutions.

6.	Hankel Transform of the First Form $(q;r)$-Dowling Numbers	Roberto B. Corcino	2021	To develop the Hankel transform of the generalised q-exponential polynomial	Hankel transform	Cigler function	Significantly, the result revealed that the Hankel transform of the first form $(q;r)$-Dowling numbers is obtained as a special case.
7.	The Hankel Transform in n-dimensions and Its Applications in Optical Propagation and Imaging	Colin J.R. Sheppard, Shan S. Kou, and Jiao Lin	2015	To examine the hyperspherically symmetric Fourier transform in (n)-dimensional space.	Hankel transform	Fourier transform	The study demonstrates that in several wave-based imaging and signal-processing contexts, Fourier representations are geometrically constrained in reciprocal space.
8.	A study on Hankel transform and its relation to the Fourier transform	Parmod Kumar	2015	To demonstrate that the Hankel transform represents an arbitrary function	Hankel transform	Fourier and Bessel transform	The key finding is that the Hankel transform can be interpreted as a continuous analogue of the Fourier-Bessel series.
9.	Application of the generalised shift operator to the Hankel transform	Natalie Baddour	2014	To demonstrate that the Hankel transform can be expressed solely in terms of the radial coordinate.	Hankel transform	Generalised-shifted function	The study establishes that the generalised shift is essential for preserving the standard operational properties of the Hankel transform.
10.	On the Laplace Transform of the Fréchet Distribution	K. A. Penson and K. Gorska	2014	To derive exact analytical expressions for the Laplace transform of the Fréchet distribution.	Laplace transform	Frechet distribution	The key findings show that closed-form Laplace transforms exist only for rational shape parameters and can be expressed via Meijer G functions.
11.	Two-Dimensional Fourier Transforms in Polar Coordinates	Natalie Baddour	2011	To develop the Fourier operational toolset of Dirac delta, exponential, and spatial shift	Two-Dimensional Fourier Transforms	Hankel Transform Radially Symmetric Functions	This study considered the polar-coordinate version of the standard 2D Fourier transform and derived the operational toolset required for standard Fourier operations.
12.	Hankel Convolution Operators on Spaces of Entire Functions of Finite Order	Jorge J. Betancor	2005	To explore and evaluate the Hankel transforms and convolution operators.	Hankel transforms and Hankel convolution operators	On Spaces of Entire Functions of Finite Order	The study revealed that Hankel transforms and Hankel convolution operators can be defined on spaces of entire functions of finite order and their duals.
13.	Hankel convolution operators on entire functions and distribu-	M. Belhadj and J.J. Betancor	2002	To explore Hankel convolution operators on the space of	Hankel convolution operators	On the space of even and on Schwartz	The study characterises the Hankel convolution operators as those that commute with

	tions			even, entire functions, and on Schwartz distribution spaces.		distribution spaces	Hankel translations and with a Bessel operator.
14.	On Hankel transforms of generalised Bessel matrix polynomials	Mohamed Abdalla	2021	The study deals with the evaluation of the Hankel transform involving Bessel matrix functions in the kernel.	Hankel transform	Bessel matrix functions as kernels	Key findings showed that the derived transforms are associated with products of certain elementary functions and generalised Bessel matrix polynomials.
15.	Beurling Distributions and Hankel Transforms	Mohamed Belhadj and Jorge J. Betancor	2002	To study the Beurling-type distributions involving Hankel transforms.	Hankel transforms	Beurling distributions (BDs)	The study established some algebraic properties for the distributional Hankel convolution.
16.	Hankel Transformation and Hankel Convolution of Tempered Beurling Distributions	M. Belhadj and J. J. Betancor	2001	To study the Hankel transform using the Fréchet spaces of tempered Beurling distributions.	Hankel transform and Hankel convolution	Tempered Beurling distributions	The study establishes that the Hankel transform can be extended as a continuous linear operator on tempered Beurling distributions.
17.	The Hankel Transform	Robert Piessens	2000	To examine the Hankel transform, an integral transform characterised by Bessel function kernels.	Hankel transform	Bessel functions/transforms	The key findings indicate that applying the Hankel transform to Laplace's partial differential equation expressed in cylindrical coordinates reduces the problem to an ordinary differential equation.

Table footnote: BDs - Beurling distributions; Fractional Hankel transform (FrHT); G-SILTM - G-S Inverse Laplace Transform Method; G-S - Gaver-Stehfest; HT – Hankel Transform; LTACA - Linear Transformation Accelerated Convergence Algorithm.

Table 5 shows classical Hankel transforms dominate the 17 studies, while advanced extensions remain underexplored, highlighting methodological opportunities.

3.2. Existing Hankel Integral Transforms and Hankel Mixtures

This chart summarises the major Hankel-transform formulations identified in the selected literature in-line with RQ1 & RQ3.

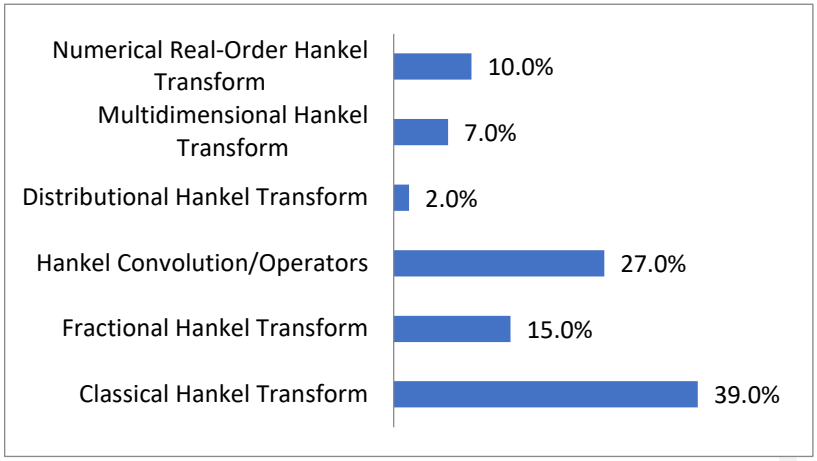

Figure 5. Distribution of Hankel Transform Variants Identified in Selected Studies

3.3. Other Existing Integral Transforms used for Oscillatory Series Characterisations

This chart identifies competing or complementary transforms used alongside Hankel methodologies in-line with RQ2.

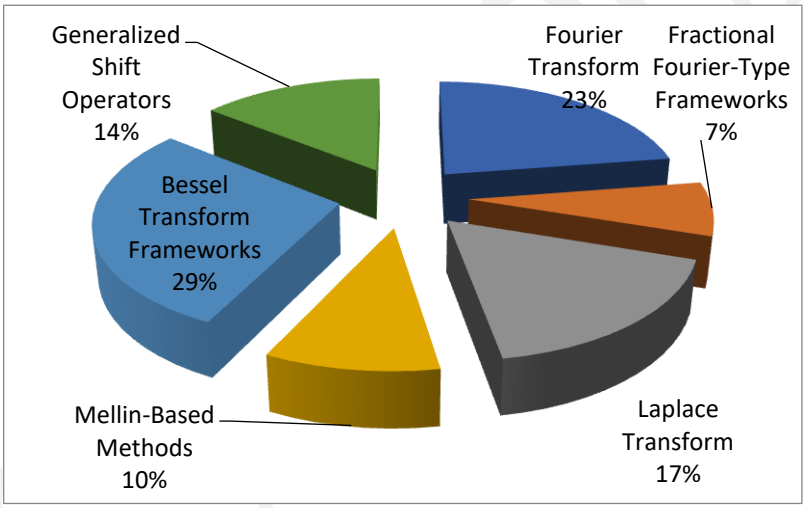

Figure 6. Alternative Integral Transforms and Related Frameworks

Figure 6 shows Fourier and Bessel approaches dominated excluded studies, while heavy-tailed transform integration remained scarce, revealing pharmacokinetic research gaps.

3.4. Limitations of Existing Hankel Transforms and Mixtures

In line with RQ4, this review synthesised methodological limitations identified across 1,388 screened records on the Hankel integral transform. Most selected studies were predominantly theoretical, emphasising transform properties, inversion formulas, convergence analysis, and operator extensions, with limited real-world applications. Only two studies incorporated heavy-tailed distributions, namely the Fréchet-based Laplace transform approach of Penson and Górska (2014) and the Hankel transformation of tempered Beurling distributions by Belhadj and Betancor (2001). Applications in pharmacokinetics and biomedical systems were notably scarce. Furthermore, existing transform-based methods primarily model average system behaviour, providing limited treatment of extreme oscillatory events. The review also revealed a lack of integration between oscillatory modelling and extreme-value theory, which have largely evolved as separate research domains

(Baddour, 2014; Baddour, 2011; Ramos *et al*., 2019). Although Fréchet distributions are widely applied in several disciplines, their incorporation into integral-transform theory, particularly Hankel-type transforms, remains limited. Consequently, the absence of Hankel-Fréchet integration and pharmacokinetic applications strongly justifies further research in this area.

3.6. Identified Gaps in the Literature

This SLR demonstrates a steady expansion of the field from foundational transform theory toward fractional, computational, distributional, and generalised formulations. Despite this growth, no study integrates Fréchet-based extreme-value modelling within a Hankel-transform framework for pharmacokinetic applications, highlighting a major research gap. Consequently, there is currently no well-established theoretical framework describing the Hankel transform of one-parameter or two-parameter Fréchet distributions, nor their associated mathematical properties, inversion structures, or applications to oscillatory systems. Collectively, these gaps demonstrate that current literature has not adequately addressed the intersection of three important areas: Hankel transform theory, Fréchet extreme-value distributions, and oscillatory pharmacokinetic processes. The absence of a unified framework linking these domains represents a substantial knowledge gap. Therefore, developing the Hankel transform of one-parameter and two-parameter Fréchet distributions, investigating their mathematical properties, and exploring their applicability to oscillatory pharmacokinetic phenomena constitute a novel and meaningful contribution to both transform theory and applied statistical modelling.

3.7. Potential Threats and Study Limitations

Two major threats to validity were data extraction errors and publication bias. These were minimised through predefined search strategies, eligibility criteria, PRISMA procedures, and manual reference screening. However, limitations remain, including reliance on English-language, peer-reviewed studies (2000-2025), exclusion of grey literature and low-quality studies, possible inadequacies in data synthesis despite established criteria, and the potential omission of relevant unpublished, emerging, or non-indexed studies on Hankel transforms and oscillatory phenomena.

4. Conclusion

This SLR synthesised developments in Hankel integral transform theory and applications from 2000-2025. Findings showed major advances in transform formulations, inversion methods, convolution structures, fractional extensions, and generalised function spaces. However, research remained largely theoretical, with limited practical applications. A significant gap exists in integrating Hankel transforms with heavy-tailed distributions, particularly the Fréchet distribution, despite its importance in modelling extreme events. Pharmacokinetic and biomedical applications were also scarce, with most studies focusing on average rather than extreme oscillatory behaviour. The review concludes that integrating Hankel transforms, Fréchet distributions, and extreme-value modelling offers a strong theoretical foundation for advancing pharmacokinetic and oscillatory process modelling.

Data Availability

The corresponding author will make available any form of data concerning this study review upon request. All database records were imported into Rayyan, duplicates removed, and titles and

abstracts screened using predefined eligibility criteria. Potentially eligible studies underwent full-text assessment, with inclusion decisions and exclusion reasons documented to ensure transparent, auditable, and reproducible study selection.

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