Weibull and Generalized Weibull Distributions: A Systematic Literature Review

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Abstract

The Weibull distribution has long been a cornerstone of reliability and survival analysis due to its mathematical simplicity and effectiveness in modelling monotonic hazard rates. However, its rigid structure often fails to capture the complex, non-monotonic failure patterns such as bathtub or unimodal shapes frequently observed in modern biomedical, engineering, and environmental data. This paper presents a systematic literature review (SLR) aimed at identifying and analysing the diverse generalisations and extensions of the Weibull distribution developed to address these limitations. Following PRISMA guidelines, a thorough screening of 1,665 records across four major databases (PubMed, Google Scholar, ScienceDirect, and Springer) was carried out, resulting in the selection of 29 key studies for detailed synthesis. The review categorises various Generalised Weibull families such as Exponentiated Weibull, Generalised Weibull, Additive Weibull and so on, including those derived through parameter addition, mixing with baseline functions like Kumaraswamy Weibull, Beta Weibull, Gamma Weibull, Burr Weibull, Fréchet Weibull, and various transformation techniques. The findings highlight that these generalised models significantly enhance modelling flexibility, offering superior goodness-of-fit for real-world datasets compared to classical sub-models. Furthermore, the study identifies critical underlying assumptions and current research gaps, providing a roadmap for future investigations into sophisticated survival analysis, epidemiology, and risk management. It is, however, recommended that bivariate and multivariate extensions be considered, while further studies should be carried out on the application of the Generalised Weibull family.

Keywords: Weibull distribution; Generalised Weibull family; Systematic literature review (SLR); Survival analysis.

1. Introduction

Among the parametric models, the Weibull distribution has historically served as one of the most widely applied due to its interpretability, computational tractability, and capacity to model monotonic hazard functions (Imran *et al*., 2024; Gómez *et al*., 2023; Sadiq *et al*., 2022; Sadiq *et al*., 2023a). However, contemporary empirical data, particularly biomedical, engineering, and environmental datasets, frequently demonstrate hazard shapes more complex than the strictly increasing or decreasing forms captured by the classical Weibull model (Suleiman *et al*., 2025; Sadiq *et al*., 2025; Sadiq *et al*., 2024). These empirical patterns often include bathtub-shaped hazards, unimodal hazards, and non-monotonic trajectories that are biologically or mechanically plausible in many real-world processes (Liu & Ananda, 2023; Kayid, 2022; Sadiq *et al*., 2025). To address these complexities, the Generalized Weibull Distribution (GWD), also known as the Extended Weibull, Flexible Weibull, or Exponentiated Weibull Family (depending on its specific parameterization), has emerged as a powerful generalization capable of capturing a wide spectrum of hazard dynamics (Selim & Salem, 2023; Dawlah & Alghamdi, 2025; Mudholkar & Srivastava, 1993; Gupta & Kundu, 2001). Unlike the traditional Weibull model that employs a scale and shape parameter, the GWD incorporates multiple shape parameters that together regulate the curvature and modality of the hazard function (Sulewski & Drapella, 2025; Pal *et al*., 2006; Sadiq *et al*., 2023b; Sadiq *et al*., 2023c).

This additional flexibility allows the GWD to represent increasing, decreasing, bathtub-shaped, J-shaped, and unimodal hazard functions within a unified modelling framework (Nassar *et al*., 2025; Shama *et al*., 2023). Historically, extensions of the Weibull distribution have been motivated by the need to overcome the rigid monotonicity inherent in the conventional Weibull hazard structure. Various parametric families, such as the generalised gamma, log-normal, exponentiated Weibull, beta-Weibull, and Kumaraswamy-

Weibull distributions have been proposed to allow richer hazard shapes (Aryal & Tsokos, 2011; Cooray, 2006). Among these, the GWD is particularly attractive because it preserves many desirable inferential characteristics, including closed-form survival and hazard functions, while significantly expanding modelling flexibility (Cordeiro *et al*., 2010). The primary objective of this systematic literature review is to comprehensively identify and analyse scholarly records of Weibull and Generalised Weibull probability distributions. The review aims to pinpoint studies directly related to the Weibull distribution, encompassing both its theoretical advancements and practical applications. Through this systematic approach, the review intends to highlight existing research gaps and suggest avenues for future investigations, drawing insights from the literature on Weibull, Generalised Weibull, and other related probability distributions.

2. Materials and Methods

This review adopted a structured methodology to ensure comprehensive analysis. The flowchart in Fig. 1 outlines the review procedures undertaken to address the research questions. These procedures are categorised into four distinct phases: research question formulation, search methodology, literature screening and research synthesis. In the initial phase, we developed specific research questions aligned with the study's overarching aim. The second phase involved designing a search strategy tailored to these questions, encompassing the selection of search terms and the strategies for identifying relevant literature. The third phase focused on data extraction and evaluating resources against predefined quality assessment criteria. Finally, the fourth phase centred on research synthesis, which included the final selection of studies for inclusion and determining any necessary subsequent actions.

2.1. Research Questions

This SLR aims to examine the underlying assumptions and empirical facts concerning the Weibull family of distributions, their Generalizations, additional parameters, and to identify possible research gaps in existing literature. Accordingly, three (3) research questions (RQs) have been formulated; they are as follows:

i. What are the existing Weibull and related distributions?
ii. What are the forms of the existing Generalised Weibull family of distributions?
iii. What are the specific functions, underlying assumptions and limitations of these Weibull, generalised Weibull, and their related distributions?

These RQs are interconnected, collectively forming the foundation of this study, and were examined simultaneously.

2.2. Search Methods

As illustrated in Figure 1, the search design delineated specific search terms, the literature database consulted, and the search procedure. The identified literature was subsequently retrieved and imported into Rayyan software for further screening and evaluation.

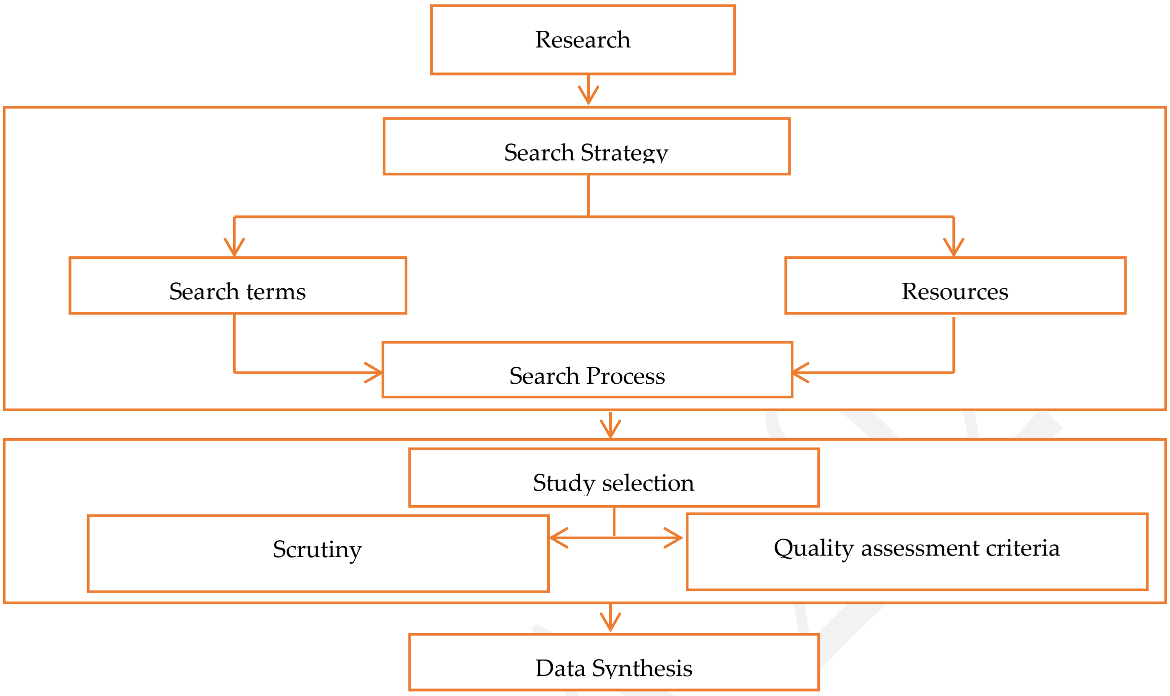

Figure 1. Algorithm for the stages of the review protocol (Azihaiwe-Justin *et al*., 2025; Ani *et al*., 2026; Ani *et al*., 2025)

2.2.1. Search Query

The search terms, as shown in Table 1, were formulated using the following guidelines described in (Haddaway *et al*., 2022; Lichtenstein *et al*., 2008), which involve the following:

i. Primary terms were directly derived from the formulated research questions (RQs)
ii. Synonyms and alternative spellings for the primary terms were identified to ensure comprehensive coverage.
iii. Relevant keywords from pertinent books and academic papers were also incorporated.
iv. Boolean operators, specifically "OR" and "AND," were utilised to combine synonyms and different spellings effectively.
v. Truncation techniques were applied to maximise the utility of search terms and broaden the search results.

Table 1: Search terms

(Weibull*) AND ((Generalized OR Extend* OR Flexible OR Exponent*) OR (Survival* OR survival Anal*) OR (Regress* OR “Regression model*”) OR (Censor*) OR (Proportional* OR Proportional Hazard Model*))

Table 1 outlines the resulting search terms

2.2.2. Database record

This study relied on secondary data, identified and sourced using the search query presented in Table 1. Literature was gathered from several prominent databases, including PubMed, Google Scholar, ScienceDirect, and Springer. The search primarily targeted the titles and abstracts within these databases to locate relevant peer-reviewed journal articles, book chapters, conference proceedings, and workshop/symposium papers.

A systematic literature review necessitates an extensive search of pertinent literature from key sources related to the topic. The search procedures implemented in this study involved several key steps (Fig. 2). As detailed in Section 2.2.2, four electronic databases were utilised. A comprehensive and exhaustive search was executed across these platforms, and the resulting literature was compiled as potential resources. Furthermore, an additional search aligned with the RQs was conducted by reviewing the reference lists of the initially identified prospective studies, leading to the discovery of further relevant records.

2.3. Literature Screening

The study selection process followed the PRISMA guidelines, as illustrated in the systematic flowchart (Figure 1). The initial identification phase yielded a total of 1,665 records across four primary databases: PubMed (n=413), Google Scholar (n=910), ScienceDirect (n=215), and Springer (n=127). Prior to the screening stage, 1,003 records were removed for various reasons. These included 723 duplicate entries, 181 records flagged as ineligible by automated screening tools, and 99 records excluded due to other factors such as language constraints or non-peer-reviewed formats. This left 662 unique records for the title and abstract screening phase.

During the title and abstract screening, 633 records were excluded as they did not align with the inclusion criteria concerning population, intervention, or outcomes. This resulted in 29 reports progressing to the next stage. It is important to note that for the retrieval and eligibility phases, the process is recorded as "NA" (not applicable), indicating that full-text reports were not formally sought, retrieved, or assessed through a conventional full-text review.

Instead, the 29 studies identified during the title and abstract screening were directly included in the final review. While this streamlined approach allowed for a rapid synthesis of the literature, it represents a deviation from the standard systematic review methodology, which typically requires a full-text assessment to minimise bias and ensure a rigorous selection process. The final review thus includes 29 studies, with the acknowledgement that inclusion was determined solely based on title and abstract screening.

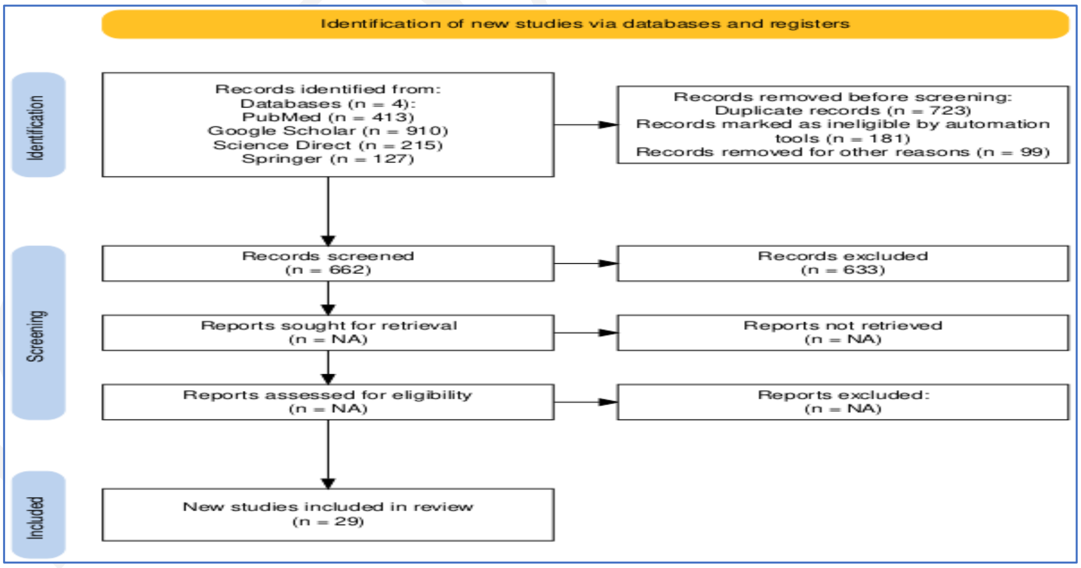

Figure 2. PRISMA flow chart

The evaluation of selected literature involved a scoring/weighting system based on these CAQs to identify studies capable of addressing the research questions. Each formulated question offers three possible responses: "yes," "partially," or "no." These responses are scored as follows: "yes = 1," "partially = 0.5," and "no = 0." A study's total quality score is calculated by summing the scores obtained for each respective question. The quality evaluation process was rigorously implemented. Any discrepancies regarding a study's quality score were resolved through detailed discussions among the authors until a unanimous agreement was reached. To ensure the reliability of the findings, only relevant studies that met a tolerable quality threshold were considered. Specifically, a study required a score greater than 2 to be selected for inclusion.

Table 2: Eligibility and screening criteria.

S/N	Inclusion	Exclusion
1.	Research published between January 1, 2000 and June 31, 2026	Grey literature, comprising works without full bibliographic information (e.g., date, type, volume, or issue), was excluded.
2.	Research articles published in English/or publications available in English	Publications reported in non-English languages
3.	Studies that focus on Weibull, GW, and related distributions	Duplicate resources (i.e. only the most recent and complete document will be chosen, while others are excluded)
4.	All studies that can address one or more of the RQs	Studies that are not related or linked to one or more of the RQs
5.	Studies originating from all countries	Studies based on self-reported data rather than objective measurements or not subjected to peer review.

Critical appraisal questions (CAQ)

i. Is the primary focus of the study clearly stated?
ii. Is the Weibull distribution, its mixture or generalisation clearly stated?
iii. Is the Weibull distribution, its mixture or generalisation used on experimental or simulated data?
iv. Are the assumptions or properties as well as the density behaviour clearly stated?

2.4. Research Synthesis

Data synthesis within a Systematic Literature Review (SLR) entails evaluating and summarising evidence from the selected studies to provide robust answers to the research questions. For this synthesis, the 29 studies that met our Critical Appraisal Questions (CAQs) were subjected to an in-depth examination, with a thorough review of each study's content. To ensure accuracy and precision, the selected studies were carefully harmonised, representing the core objective of this review. The extracted data covered both qualitative and quantitative information.

The data synthesis process was conducted as follows: Data for RQ1 and RQ2 were systematically organised utilising various visual tools. For RQ3, the specific functions, assumptions, and limitations of the Weibull and related distributions were extracted from the selected studies and summarised in tabular format. The content evaluation measures outlined previously served as a valuable guide throughout the data synthesis process.

Table 3: Description of eligible literature

S/N	Authors & year	Paper title	Type of Data	Area of Application	Form of Weibull distribution developed	Findings /Remarks
1.	Ishaq *et al*., (2017)	Weibull-Kumaraswamy distribution theory and application	Real Data	General Statistical Modelling	Weibull-Kumaraswamy distribution	Effectively captured skewness/kurtosis; outperformed competing models

2.	A. Yakubu and Sani I. Doguwa (2017)	On the Properties of the Weibull-Burr III Distribution	Censored & uncensored	Survival Analysis	Weibull-Burr III Distribution	Compound distribution fits real data better than competitors in both contexts.
3.	Ishaq *et al.*, (2017)	Weibull-Burr type X distribution: its properties	Real Data	General Statistics	Weibull-Burr type X distribution	Four-parameter model with smaller AIC/AICC values than compared models.
4.	Usman *et al.*, (2022)	Mixture-cure rate model for right-censored….	Clinical Trials / Colorectal Cancer	Medical	Weibull exponentiated exponential	Superior fitting for right-censored data; outperformed traditional sub-models.
5.	Yakubu *et al.*, (2024)	The Weibull exponentiated exponential cure fraction model for right-censored survival data…….	Medical / Cancer Data	Medical	Weibull exponentiated exponential	Strong competitor for long-term survival analysis with high precision.
6.	Usman *et al.*, (2020)	Inference on Weibull exponentiated exponential distribution	Real-world data	General modeling	Weibull exponentiated exponential	Provided significant mathematical flexibility and effective analysis of real data.
7.	Abdulfatai *et al.*, (2025)	A Weibull-Fréchet Proportional Hazard model	Tuberculosis Patients	Medical	Weibull-Fréchet	Identified age, smoking, and HIV as significant mortality predictors
8.	Ishaq *et al.*, (2022)	A new Odd F-Weibull distribution properties	Monthly Exchange Rate	Actuarial / Finance	Odd F-Weibull distribution	Robust alternative to Beta-Weibull; provided superior fit for financial data.
9.	Ishaq *et al.*, (2019)	Weibull Burr X – Generalised family of distributions	Real Data	General Modeling	Weibull Burr X – Generalised	High flexibility and accuracy; powerful alternative to standard families.

10.	Abonongo, *et al.*, (2024)	Exponentiated generalised Weibull exponential (EGWE)	Real Data	General life science	Exponentiated generalised Weibull exponential	Captured diverse hazard shapes like bathtub and J-shaped curves
11.	Elgarhy *et al.* (2017)	Exponentiated Weibull-exponential distribution	Real Data	General Modeling	Exponentiated generalised Weibull exponential	New four-parameter model with explicit expressions for entropies.
12.	Mastor *et al.*, (2022)	The Extended Exponential Weibull Distribution	Real Data	General Modeling	Extended exponential Weibull	Developed via LAII technique; validated through Monte Carlo simulations.
13.	Usman *et al.*, (2020)	Inference on the Weibull exponentiated exponential distribution and Applications	Real Data	General Modeling	Weibull exponentiated exponential distribution	Observed to be more flexible than standard Weibull and exponential models.
14.	Cordeiro *et al.*, (2017)	The exponentiated Weibull-H family of distributions	Real data	Survival Analysis	Exponentiated Weibull - H distribution	Proven flexibility of the generated family through incomplete moments.
15.	Abdullah *et al.*, (2015)	Inference from the exponentiated Weibull model	Real Data	General modeling	Exponentiated Weibull model	Bayesian framework offered superior fit for forecasting future responses.
16.	Cordeiro *et al.*, (2014)	The exponential-Weibull lifetime distribution	Real Data	Survival Analysis	Exponentiated Weibull	Flexible three-parameter model using Meijer's G functions for moments.
17.	Abdullah *et al.*, (2024)	Inference on Weibull inverted exponential.	Engineering Data	Industrial Engineering	Weibull	Robust framework for parameter estimation in life testing scenarios
18.	Ibrahim and Wikanda. (2026)	A flexible Nakagami-Weibull mixture cure model	Melanoma Dataset	Medical	Nakagami-Weibull	Effectively captured complex cure fractions in long-term survivors
19.	Izomo *et al.*, (2025)	Hybrid distribution of WEG and	Real-life data	General modeling	WEGD	Evaluated via Kaplan-Meier; showed high

		Gamma (WEGD)				robustness across replications.
20.	Ishaq A. *et al*., (2024)	The Exponentiated Weibull Proportional Hazard Regression model with application to censored survival data.	Real-life data	Medical	Exponentiated Weibull model	Powerful tool for analysing complex survival data
21.	Sanku *et al*., (2017)	A New Extension of Weibull Distribution with Application to Lifetime Data	Real data	Survival Analysis	Extension of Weibull Distribution	Outperformed standard Weibull; modelled bathtub/upside-down curves.
22.	Kahadawala (2006)	Extension of Weibull Distribution: the odd Weibull family	Uncensored data	Reliability / Environmental	Extension of Weibull Distribution	Useful for testing goodness-of-fit in the presence of censoring
23.	Hoang and Lai. (2007)	On Recent Generalisations of the Weibull Distribution	Theoretical	Reliability Modeling	Generalisations of the Weibull Distribution	Provided an overview of interrelationships and future research directions.
24.	Rangoli *et al*., (2024)	A New Modified Power Generalised Weibull Distribution	Hospital dataset	Medical	Generalised Weibull Distribution	Captured bimodal densities; Bayesian methods gave superior precision.
25.	Muhammad *et al*., (2024)	The development of an extended Weibull model with applications to medicine, industry and actuarial sciences	Medical / Industrial / Actuarial	Risk Management	Generalized	Highlighted utility for Value at Risk and Expected Shortfall analysis
26.	Mustafa, *et al*.,	Modified generalised	Real-life time data	Reliability Studies	Generalized	Addressed bathtub-shaped hazards with

	(2023)	Weibull distribution				superior precision.
27.	Cordeiro *et al*., (2015)	A new generalised Weibull family of distributions	Censored survival data	General modeling	Generalized	Extended Normal/Gamma/Gumbel; introduced new regression models
28.	Alsulami and Alghamd. (2025)	A New Extended Weibull Distribution (NEW)	Real data	Engineering / Physics	Generalized	Remarkable flexibility via logarithmic transformation; effective ML estimation
29.	Afify *et al*. (2025)	New flexible Weibull (GKMW) distribution	Real data	Lifetime Data Analysis	Generalized	Highly adaptable model for bathtub and J-shaped hazard patterns.

3. Results and Discussions

The systematic review of 29 peer-reviewed studies, extending across various methodologies and fields, reveals a significant shift in survival analysis toward the use of Generalised Weibull families. The synthesis of results is organised around the three core research questions: the identification of existing related distributions, the forms of generalisation, and the evaluation of their performance across diverse fields.

3.1 Study Selection and PRISMA Flow Analysis

The systematic review of 29 peer-reviewed studies reveals a significant shift in survival analysis toward the use of Generalised Weibull families. The selection was conducted through a rigorous four-stage process as delineated by the PRISMA guidelines. This structured approach ensured that the final synthesis was based on high-quality, peer-reviewed literature relevant to the research questions.

The search was conducted across four primary electronic databases, which yielded a total of 1,665 records. The distribution of these records was as follows: Google Scholar: 910 records, PubMed: 413 records**,** ScienceDirect: 215 records and **S**pringer: 127 records. Before the formal screening began, 1,003 records were excluded. This significant reduction was primarily due to 723 duplicate entries that were identified as redundant across databases; 181 records were flagged by automated tools as not meeting basic scholarly criteria, and 99 records were removed due to language barriers (non-English) or non-peer-reviewed formats. 662 unique records were left for title and abstract screening.

During this stage, the researchers applied strict inclusion and exclusion criteria (as detailed in Table 2). A total of 633 records were excluded at this point because they did not align with the study's specific focus on Weibull generalisations, population interventions, or relevant outcomes. Following the screening of titles and abstracts, 29 reports were identified as meeting all quality assessment criteria. Interestingly, this review utilised a streamlined protocol where these 29 studies were directly included in the final synthesis after the abstract screening stage. This resulted in a final corpus of 29 high-quality papers that form the basis of the theoretical and practical analysis in this review.

3.2 Synthesis of Distributional Forms and Generalisations

The literature highlights that those generalisations of the Weibull distribution primarily occur through three mechanisms: parameter addition, transformation, and mixing.

Generalised Families: A significant majority of the studies (e.g., Ishaq *et al.*, 2017; Usman et al., 2020; Dawlah *et al*., 2025) focus on adding shape parameters to the baseline Weibull. This is often achieved through generator families like the Weibull-G or Odd F-Weibull, which allow the resulting models to capture complex hazard shapes such as bathtub, J-shaped, and unimodal curves.

Mixed and Hybrid Models: Studies such as Usman *et al*. (2022) and Abdullahi *et al.* (2026) utilise mixture cure rate models. These are particularly effective in clinical settings where a portion of the population (the "cure fraction") is non-susceptible to the event of interest. Hybrid models, like the WEGD (Izomo et al., 2025), combine Weibull, Exponential, and Gamma properties to enhance robustness.

Across all 29 studies, the performance and Goodness-of-Fit of the generalised models consistently outperformed the classical two-parameter Weibull and its immediate sub-models.

Statistical Criteria: The findings frequently cite lower values in discrimination criteria such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). For instance, the Weibull-Burr type X and EGWE distributions demonstrated superior fit for real-world datasets compared to traditional models (Ishaq *et al*., 2017; Anuwoje *et al.*, 2024).

Hazard Flexibility: The standard Weibull is limited to monotonic hazard rates. However, the reviewed literature confirms that generalised versions like the New Exponentiated Weibull (NEW) and the Generalised Kavya-Manoharan Weibull (GKMW) distribution successfully model non-monotonic behaviours, which are critical for accurately describing the "bathtub" failure patterns in engineering and medicine.

3.3 Practical Applications and Predictors

The review identifies that these models are not merely theoretical but have profound practical utility:

i. Medical Applications: In tuberculosis and cancer research, generalised models like the Weibull-Fréchet PH model have identified critical predictors of mortality, including age, smoking history, and HIV status, with higher precision than standard Cox models (Lawal *et al.*, 2025).

ii. Actuarial and Industrial Utility: Extensions like the Complementary Bell Weibull have proven useful in risk management for calculating Value at Risk (VaR) and Expected Shortfall, while inverted Weibull models have improved parameter estimation in industrial life testing under censoring (Imran *et al*., 2024; Fathi *et al.*, 2024).

3.4 Discussion of Research Gaps

While the generalisations provide immense flexibility, the discussion reveals several recurring challenges:

1. Computational Complexity: As models move from two to four or five parameters, estimation via Maximum Likelihood (MLE) becomes numerically intensive, often requiring specialised software like R or complex algorithms like EM and Newton-Raphson.

2. Censoring Sensitivity: Many models show high sensitivity to different types of censoring (right, progressive, or first-failure), suggesting that while they fit the data well, their predictive stability requires further validation in small-sample contexts.

3. Bayesian vs. Classical Inference: There is an emerging trend toward Bayesian frameworks (Khan *et al.,* 2015; Rangoli *et al*., 2025), which often yield lower standard errors than classical MLE, yet these methods remain less common in the broader literature reviewed.

Table 4: Existing Weibull, the generalisation, and the mixing distributions

S/N	Baseline Functions	Mixture	General Assumption	Limitations	Source(From Review)
1.	Weibull	Weibull, lognormal, exponential	Constant Hazard rate	Monotonic Hazard rate	Hoang Pham *et al.* (2007); Kahadawala Cooray (2006
2.	Exponential	Weibull	Constant Hazard rate	Lack of flexibility	Usman *et al.* (2020); Elgarhy *et al.* (2017)
3.	Gamma	generalized	Flexible	Parameter interpretation	Izomo *et al*. (2025); Cordeiro *et al*. (2015)
4.	Log-Logistic	generalized	Suitable for modelling	Can't adapt to complex patterns	Ishaq *et al*. (2022); Cordeiro *et al*. (2015)
5.	Fréchet	Weibull	Data represent maxima from i.i.d. variables	Specialised for extremes, not general data	Lawal *et al.* (2025); Afify *et al*. (2016
6.	Kumaraswamy	Generalized	Data is bounded within the unit interval (0,1)	Restricted to the (0,1) interval	Ishaq et al. (2017)
7.	Exponentiated	Alpha-Power	Baseline Distribution Validity:	Increased Complexity	Anuwoje *et al.* (2024),Sanku *et al.* (2017)
8.	Rayleigh	Exponentiated	Underlying Normal Component	Limited Flexibility	Sanku *et al*. (2017); Dawla

3.6 Limitations Affecting the Validity of the Review

This section identifies two primary risks to the review's validity: publication bias and study limitations in data extraction. To reduce these, the researchers strictly followed a structured search protocol guided by the PRISMA framework. They also acknowledged that digital searches of titles and abstracts might overlook key literature. To address this, the team manually scrutinised the reference lists of all shortlisted papers to uncover any relevant studies missed initially. This double-layered approach ensures a more comprehensive analysis while significantly bolstering the overall reliability of the findings.

4. Conclusion

This systematic literature review underscores the evolution of the Weibull distribution from a foundational monotonic model to a highly flexible family of generalised distributions. By synthesising evidence from 29 crucial studies, we have demonstrated that the integration of additional shape parameters and the hybridisation with other baseline functions such as the Exponentiated Exponential, Burr, and Nakagami distributions effectively resolve the inherent inflexibility of the classical Weibull framework. These advancements allow researchers to accurately model complex hazard trajectories, including bathtub, J-shaped, and bimodal patterns, which are essential for precise analysis in medicine, engineering, and actuarial sciences.

The results consistently show that generalised models like the Weibull-Kumaraswamy and the Generalised Kavya-Manoharan Weibull distribution outperform traditional models in terms of log-likelihood and information criteria across diverse real-world applications. However, this increased flexibility often introduces greater computational complexity and challenges in parameter estimation, particularly under censoring. Moving forward, future research should focus on bridging these computational gaps, exploring bivariate and multivariate extensions, and refining Bayesian frameworks to enhance predictive accuracy. Ultimately, the transition toward these generalised families represents a significant leap in our ability to interpret and forecast complex survival phenomena in increasingly data-driven fields.

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