LOGARITHMIC RATIO-PRODUCT ESTIMATOR OF POPULATION VARIANCE IN SIMPLE RANDOM SAMPLING SCHEME

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ABSTRACT

Estimating population variance is a significant challenge in sample survey techniques. Current estimators often lack sufficient precision, as highlighted by various researchers. This study developed a logarithmic ratio-product estimator for population variance. The bias and mean squared error (MSE) of the developed estimator were derived up to the first-order approximation, along with optimal values. Theoretical and empirical comparisons with other ratio estimators demonstrate the superiority of the new estimator, achieving a lower MSE (1,505,349.49 and 1,187,207) and a higher percentage relative efficiency (445.3406 and 609.4563) for Populations 1 and 2, respectively. Consequently, the developed estimator proves more effective than existing alternatives and is recommended for practical applications.

Keywords: Logarithmic Ratio-Product Estimator, Population Variance, Bias, Mean Square Error, Simple Random Sampling Scheme.

1. INTRODUCTION

The use of auxiliary information to improve estimates of population parameters of the study variable is well-known in sample surveys. The ratio-type estimators are preferred when there is a highly positive linear relationship between the study and auxiliary variables described by a straight line that passes through the origin. Regression-type estimators are preferred if the straight line does not pass through the origin. When there is a highly negative linear relationship between the study and auxiliary variables with the regression line passing through the origin, the product-type estimators are most preferable. Cochran (1940) introduced the customary ratio-type estimator, while Murthy (1964) proposed the customary product-type estimator. Srivastava (1967) discussed a power transformation estimator for which the customary ratio and product-type estimators are special cases. The use of auxiliary information in form of ratio and product-type estimators was initiated by Bahl and Tuteja (1991). Many other scholarly works on the use of auxiliary variables exist in literature Kadilar and Cingi (2004), Khoshnevisan et al. (2007), Onyeka (2012), Chaun and Singh (2014), Yadav et al. (2014), Onyeka et al. (2015), Etebong, P.C. (2016), Subramani and Ajith (2016), Madhulika et al. (2017). The present study utilizes auxiliary information to improve estimates of the population variance of the study variable by introducing logarithmic ratio-product type estimators.

2. BACKGROUND OF THE STUDY

Consider a finite population, $U=(U_1,U_2,...,U_N)$, of N units; let a sample be drawn using simple random sampling without replacement (SRSWOR); let Y and X represent the study variable and auxiliary variable respectively. For a sample survey problem, if the interest is to estimate the population variance, S_y^2, when the population and sample coefficient of variation of auxiliary variable is known, one should assume that a sample of size, *n*, is to be selected using simple random sampling scheme. Each unit of the population is identifiable by means of assigning numbers to the population units from 1 to *N*, where the numbers assigned are of nominal scale.

Some notations used in the study are given below;

- $N-$population size.
- $n-$sample size.
- $f = n/N$ $-$sampling fraction.
- $Y-$ study variable.
- $X-$ auxiliary variable.
- $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the mean per unit estimator of *X*.
- $\bar{Y}$, $\bar{X}$ $-$ population means of *Y* and *X*, respectively.
- $\bar{y}, \bar{x}$ $-$ sample means of *Y* and *X*, respectively.
- s_y^2, s_x^2 $-$ sample variance of *Y* and *X*, respectively.
- $Bias(.)$ $-$ Bias of the estimator.
- MSE$-$ Mean Squared Error of the estimator.
- PRE$-$ Percentage Relative Efficiency.
- S_Y^2 $-$ Population variance of *Y*.
- S_X^2 $-$ Population variance of *X*.
- ρ $-$ correlation coefficient between, *Y* and *X*.
- C_Y $-$ population coefficient of variation of *Y*.
- C_x $-$ population coefficient of variation of *X*.
- c_x $-$ sample coefficient of variation of *X*.
- l $-$ preliminary large sample size.
- β_1 $-$ coefficient of skewness of *X*
- β_2 $-$ coefficient of kurtosis of *X*
- Q_1 $-$First quartile of the sample
- Q_2 $-$Second quartile of the sample
- M_d $-$Median

2.1. Some Existing Estimators Relevant to the Work with their Biases and Mean Squared Errors

Isaki (1983) defined ratio-type estimator as;

$$s_I^2 = s_y^2\left(\frac{S_x^2}{s_x^2}\right). \quad (1)$$

where I means Isaki's estimator

The bias and mean squared error of s_I^2 using first degree of approximation are, respectively, given as;

$$bias\left(s_I^2\right) = \left(\frac{1-f}{n}\right)S_y^2\left\{\left(\theta_{04}-1\right)-\left(\theta_{22}-1\right)\right\} \quad (2)$$

$$MSE\left(s_I^2\right) = \left(\frac{1-f}{n}\right)S_y^4\left\{\left(\theta_{40}-1\right)+\left(\theta_{04}-1\right)-2\left(\theta_{22}-1\right)\right\}. \quad (3)$$

Solanki *et al* (2015) developed a ratio type estimator for population variance given as;

$$\hat{S}_{S1}^2 = s_y^2\left[\frac{\beta_{2(y)}S_y^2+Q_2^2}{\beta_{2(y)}s_y^2+Q_2^2}\right] \quad (4)$$

Where $\hat{S}_{S1}^2$ is a notation for Solanki's estimator.

The bias and mean squared error of $\hat{S}_{S1}^2$ using first degree of approximation are, respectively, given as;

$$bias = \left(\frac{1-f}{n}\right)S_y^2R_{S1}\left[R_{S1}\left(\theta_{04}-1\right)-\left(\theta_{22}-1\right)\right] \quad (5)$$

$$MSE = \left(\frac{1-f}{n}\right)S_y^4\left[\left(\theta_{40}-1\right)+R_{S1}^2\left(\theta_{04}-1\right)-2R_{S1}\left(\theta_{22}-1\right)\right] \quad (6)$$

Kadilar and Cingi (2006) proposed a ratio estimator for population variance as;

$$\hat{S}_{KC1}^2 = s_y^2\left[\frac{S_x^2+\beta_{2(x)}}{s_x^2+\beta_{2(x)}}\right] \quad (7)$$

where $\hat{S}_{KC1}^2$ is a notation for Kadilar and Cingi (2006) estimator.

The bias and MSE $\hat{S}_{KC1}^2$ of are respectively given as;

$$bias = \left(\frac{1-f}{n}\right)S_y^2R_{KC1}\left[R_{KC1}\left(\theta_{04}-1\right)-\left(\theta_{22}-1\right)\right] \quad (8)$$

$$MSE = \left(\frac{1-f}{n}\right)S_y^4\left[\left(\theta_{40}-1\right)+R_{KC1}^2\left(\theta_{04}-1\right)-2R_{KC1}\left(\theta_{22}-1\right)\right] \quad (9)$$

where $R_{KC1} = \frac{S_x^2}{S_x^2+\beta_{2(x)}}$

Singh et al. (2013) also introduced a ratio-type estimator for the population variance, which can be described as follows:

$$\hat{S}_{S3}^2 = s_y^2\left[\frac{S_x^2+Q_1^2}{s_x^2+Q_1^2}\right] \quad (10)$$

where S3 is a notation for Singh's estimator.
The bias and MSE of $\hat{S}^2_{S3}$ are respectively given as;

$$bias = \left(\frac{1-f}{n}\right) S_y^2 R_{S3}\left[R_{S3}\left(\theta_{04}-1\right)-\left(\theta_{22}-1\right)\right] \tag{11}$$

$$MSE = \left(\frac{1-f}{n}\right) S_y^4\left[\left(\theta_{40}-1\right)+R_{S3}^2\left(\theta_{04}-1\right)-2R_{S3}\left(\theta_{22}-1\right)\right] \tag{12}$$

where $R_{S3} = \frac{S_x^2}{S_x^2 + Q_1^2}$

The population variance's ratio estimator proposed by Kadilar and Cingi's (2006) is also given by;

$$S^2_{KC2} = s_y^2\left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right] \tag{13}$$

where $\hat{S}^2_{KC2}$ is a notation for Kadilar and Cingi's estimator.
The bias and MSE of $\hat{S}^2_{KC2}$ are respectively given as;

$$bias = \left(\frac{1-f}{n}\right) S_y^2 R_{KC2}\left[R_{KC2}\left(\theta_{04}-1\right)-\left(\theta_{22}-1\right)\right] \tag{14}$$

$$MSE = \left(\frac{1-f}{n}\right) S_y^4\left[\left(\theta_{40}-1\right)+R_{KC2}^2\left(\theta_{04}-1\right)-2R_{KC2}\left(\theta_{22}-1\right)\right] \tag{15}$$

where $R_{KC2} = \frac{S_x^2}{S_x^2 + C_x}$

Another ratio estimator for population variance was proposed by Subramani and Kumarapandiyan (2012) which is defined;

$$\hat{S}^2_{SK} = s_y^2\left[\frac{S_x^2 + M_d}{s_x^2 + M_d}\right] \tag{16}$$

where $\hat{S}^2_{SK}$ is a notation for Subramani and Kumarapandiyan (2012) estimator.
The bias and MSE of $\hat{S}^2_{SK}$ are respectively given as;

$$bias = \left(\frac{1-f}{n}\right) S_y^2 R_{SK}\left[R_{SK}\left(\theta_{04}-1\right)-\left(\theta_{22}-1\right)\right] \tag{17}$$

$$MSE = \left(\frac{1-f}{n}\right) S_y^4\left[\left(\theta_{40}-1\right)+R_{SK}^2\left(\theta_{04}-1\right)-2R_{SK}\left(\theta_{22}-1\right)\right] \tag{18}$$

where, $R_{SK} = \frac{S_x^2}{S_x^2 + M_d}$

Also, Solanki *et al.* (2012) proposed an estimator for the population variance which is defined as;

$$S^2_{S(\alpha,\delta)} = s_y^2\left[2-\left(\frac{s_x^2}{S_x^2}\right)^{\alpha}\exp\left\{\frac{\delta\left(s_x^2-S_x^2\right)}{\left(s_x^2+S_x^2\right)}\right\}\right]. \tag{19}$$

The bias and MSE of $\hat{S}^2_{(\alpha,\delta)}$ are respectively given as;

$$bias = \left(\frac{1-f}{n}\right)S_y^2\left[-\frac{(2\alpha-\delta)}{2}(\theta_{22}-1)-\frac{(2\alpha+\delta)(2\alpha+\delta-2)}{8}(\theta_{04}-1)\right] \tag{20}$$

$$MSE = \left(\frac{1-f}{n}\right)S_y^4\left[(\theta_{40}-1)+\frac{(2\alpha+\delta)^2}{4}(\theta_{04}-1)-(2\alpha+\delta)(\theta_{22}-1)\right] \tag{21}$$

where; (α,δ) are suitable chosen scalars

3. METHODS OF EFFICIENCY COMPARISON OF THE PROPOSED ESTIMATORS WITH SOME EXISTING ESTIMATORS

3.1. **Methods for Analytical Study**

The study adopts analytical method. Mathematical and statistical operations involving parameters in linear regression and ratio methods were used to develop the estimators for population variance. The concepts of first order approximation, expectation and method of partial differential equation were used to derive the bias and Mean Square Error (MSE) of the proposed estimators. The concept of inequality was also used to obtain the efficiency condition of the proposed estimators.

A new estimator say s^2_{Pi} will be more efficient than any estimator say s^2_T if,

$$\text{MSE}(S^2_{pi}) < \text{MSE}(S^2_T) \tag{22}$$

3.2. **Methods for Empirical Study**

To empirically evaluate the efficiency of the new parameter-based estimators, two life data sets are used and simulation studies was also carried out. R statistical package (version 3.5.3) was employed in analyzing the data.

The Percentage Relative Efficiency (PRE) was employed as performance measure for determining the best among the competing estimators.

The PRE of different estimators, S_T^2 with respect to S_I^2 is defined as,

$$\text{PRE}\left(S_I^2, S_T^2\right) = \frac{MSE(\mathrm{S}_I^2)}{MSE(\mathrm{S}_T^2)}\times 100 \tag{23}$$

3.3. **Proposed Estimator**

Motivated by the work of Singh and Neha-Agnihotri (2008), Abdullahi and Yahaya (2017), the proposed logarithmic ratio-product estimator for population variance is given as;

$$\hat{S}^2_{CVRP} = s_y^2\left[\log\left(\lambda_1\left(\frac{aC_x+b}{ac_x+b}\right)+(1-\lambda_1)\left(\frac{ac_x+b}{aC_x+b}\right)\right)\right] \tag{24}$$

$$where;\gamma = \frac{aC_x}{aC_x + b}$$

3.4. **The Bias and MSE of the Proposed Estimator**

Expanding (3.43) in terms of $\Omega's$ and assuming that the contribution of terms involving powers in Ω_{sy} and Ω_{sx} higher than the second is negligible, we have;

$when \quad a = M_d \quad and \quad b = \bar{X}, \quad then;$

$$\hat{S}^2_{CVRP} = S_y^2\left(1+\Omega_{sy}\right)\left[\lambda_1 \log\left(1+\gamma\Omega_{sx}\right)^{-1} + \left(1-\lambda_1\right)\log\left(1+\gamma\Omega_{sx}\right)\right] \tag{25}$$

$$= S_y^2\left(1+\Omega_{sy}\right)\left[\lambda_1\left(1-\gamma\Omega_{sx}+\gamma\Omega_{sx}^2\right)+\left(1-\lambda_1\right)\left(\gamma\Omega_{sx}+\frac{\gamma\Omega_{sx}^2}{2}\right)\right]$$

$$= S_y^2\left[\lambda_1(1+\Omega_{sy})\left(1-\gamma\Omega_{sx}+\gamma\Omega_{sx}^2\right)+\left(1-\lambda_1\right)\left(1+\Omega_{sy}\right)\left(\gamma\Omega_{sx}+\frac{\gamma\Omega_{sx}^2}{2}\right)\right]$$

$$= S_y^2\left[\lambda_1\left(1-\gamma\Omega_{sx}+\gamma\Omega_{sx}^2+\Omega_{sy}-\gamma\Omega_{sy}\Omega_{sx}+\gamma\Omega_{sy}\Omega_{sx}^2\right)+\left(1-\lambda_1\right)\left(\gamma\Omega_{sx}+\gamma\frac{\Omega_{sx}^2}{2}+\gamma\Omega_{sy}\Omega_{sx}+\frac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}\right)\right]$$

$$= S_y^2\left[\gamma\Omega_{sx}+\frac{\gamma\Omega_{sx}^2}{2}+\gamma\Omega_{sy}\Omega_{sx}+\frac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}+\lambda_1\left(\begin{array}{l}1-\gamma\Omega_{sx}+\gamma\Omega_{sx}^2+\Omega_{sy}-\gamma\Omega_{sy}\Omega_{sx}+\gamma\Omega_{sy}\Omega_{sx}^2+\\ \ldots\ldots\ldots\ldots-\gamma\Omega_{sx}-\dfrac{\gamma\Omega_{sx}^2}{2}-\gamma\Omega_{sy}\Omega_{sx}-\dfrac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}\end{array}\right)\right]$$

$$= S_y^2\left[\left(\gamma\Omega_{sx}+\frac{\gamma\Omega_{sx}^2}{2}+\gamma\Omega_{sy}\Omega_{sx}+\frac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}\right)+\lambda_1\left(1-2\gamma\Omega_{sx}+\frac{\gamma\Omega_{sx}^2}{2}+\Omega_{sy}-2\gamma\Omega_{sy}\Omega_{sx}+\frac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}\right)\right]$$

$$E\left(\hat{S}^2_{CVRP}-\hat{S}^2_y\right) = S_y^2 E\left[\lambda_1+\Omega_{sy}+\left(1-2\lambda_1\right)\left(\gamma\Omega_{sx}+\gamma\Omega_{sy}\Omega_{sx}\right)-\left(1+\lambda_1\right)\left(\frac{\gamma\Omega_{sx}^2}{2}+\frac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}\right)\right] \tag{26}$$

$$bias\left(\hat{S}^2_{CVRP}\right) = S_y^2\left(\frac{1-f}{n}\right)\left[\lambda_1+\left(1-2\lambda_1\right)\gamma\left(\theta_{22}-1\right)-\left(1+\lambda_1\right)\gamma\left(\frac{\theta_{04}-1}{2}\right)\right] \tag{27}$$

To obtain the Mean Square Error (MSE), we square both side of (25) and neglect the terms of **Ω**'s having power greater than two, we have;

$$E\left(\hat{S}^2_{CVRP}-\hat{S}^2_y\right) = S_y^2 E\left[\left[\lambda_1+\Omega_{sy}+\left(1-2\lambda_1\right)\left(\gamma\Omega_{sx}+\gamma\Omega_{sy}\Omega_{sx}\right)-\left(1+\lambda_1\right)\left(\frac{\gamma\Omega_{sx}^2}{2}+\frac{\gamma\Omega_{sy}\Omega_{sx}^2}{2}\right)\right]\right]^2$$

Taking expectation of both sides, we have;

$$MSE\left(\hat{S}^2_{CVRP}\right) = \frac{1-f}{n}S_y^4\left[\lambda_1^2+\left(\theta_{40}-1\right)+\left(1-2\lambda_1\right)^2\gamma\left(\theta_{04}-1\right)-2\lambda_1\gamma\left(\theta_{22}-1\right)\right] \tag{28}$$

3.5. Optimal MSE of the Proposed Estimator

To obtain the optimal value i.e the value of λ_1 that minimizes $MSE\left(\hat{S}^2_{CVPR}\right)$, we take partial derivative of $MSE\left(\hat{S}^2_{CVPR}\right)$ and equate to 0

$$\frac{\partial\left(\hat{S}^2_{CVRP}\right)}{\partial\left(\lambda_1\right)}=\left[S_y^4\left(\frac{1-f}{n}\right)\left\{\lambda_1^2+\Omega_{sy}+\left(\theta_{40}-1\right)+\left(1-2\lambda_1\right)^2\gamma\left(\theta_{04}-1\right)-2\lambda_1\gamma\left(\theta_{22}-1\right)\right\}\right]$$

$$\lambda_1-4\gamma\theta_{04}+4\lambda_1\gamma\theta_{04}+4\gamma-4\lambda_1\gamma-2\gamma\theta_{22}+2\gamma=0$$

$$\lambda_1+4\lambda_1\gamma\theta_{04}-4\lambda_1\gamma=4\gamma\theta_{04}-4\gamma+2\gamma\theta_{22}-2\gamma$$

$$\lambda_1\left(1+4\gamma\theta_{04}-4\gamma\right)=2\left(2\gamma\theta_{04}+\gamma\theta_{22}-3\gamma\right)$$

$$\lambda_1=\frac{2\left(2\gamma\theta_{04}+\gamma\theta_{22}-3\gamma\right)}{1+4\gamma\theta_{04}-4\gamma}$$

3.6. Efficiency Comparisons

The proposed estimator is more efficient than existing estimators considered if;

1) The MSE of the Proposed $MSE\left(\hat{S}^2_{CVRP}\right)$ estimator is less than the MSE of the Isaki (1983) ratio estimator $MSE\left(\hat{S}^2_{I}\right)$ in equation (3) i.e

$$\bullet MSE\left(\hat{S}^2_{CVRP}\right)\le MSE\left(\hat{S}^2_{I}\right)$$

$$\Rightarrow\frac{1-f}{n}S_y^4\left[\lambda_1^2+\left(\theta_{40}-1\right)+\left(1-2\lambda_1\right)^2\gamma\left(\theta_{04}-1\right)\right]\le\frac{1-f}{n}S_y^4\left\{\left(\theta_{04}-1\right)+\left(\theta_{04}-1\right)-2\left(\theta_{22}-1\right)\right\}$$

$$MSE\left(\hat{S}^2_{I}\right)-MSE\left(\hat{S}^2_{CVRP}\right)=\frac{1-f}{n}S_y^4\left[\lambda_1^2+\left(\left(1-2\lambda_1\right)^2\gamma-1\right)\left(\theta_{04}-1\right)+2\left(\theta_{22}-1\right)\right]>0$$

2) The MSE of the Proposed $MSE\left(\hat{S}^2_{CVRP}\right)$ estimator is less than the MSE of the Kadilar and Cingi (2006) ratio estimator $MSE\left(\hat{S}^2_{KC1}\right)$ in equation (9) i.e

$$\bullet MSE\left(\hat{S}^2_{CVRP}\right)\le MSE\left(\hat{S}^2_{KC1}\right)$$

$$\Rightarrow\frac{1-f}{n}S_y^4\left[\lambda_1^2+\left(\theta_{40}-1\right)+\left(1-2\lambda_1\right)^2\gamma\left(\theta_{04}-1\right)\right]\le\frac{1-f}{n}S_y^4\left\{\left(\theta_{04}-1\right)+R_{KC1}^2\left(\theta_{04}-1\right)-2R_{KC1}\left(\theta_{22}-1\right)\right\}$$

$$MSE\left(\hat{S}^2_{KC1}\right)-MSE\left(\hat{S}^2_{CVRP}\right)=\frac{1-f}{n}S_y^4\left[\lambda_1^2+\left(\left(1-2\lambda_1\right)^2\gamma-R_{KC1}^2\right)\left(\theta_{04}-1\right)+2R_{KC1}\left(\theta_{22}-1\right)\right]>0$$

3) The MSE of the Proposed $MSE\left(\hat{S}^2_{CVRP}\right)$ estimator is less than the MSE of the Kadilar and Cingi (2006) ratio estimator $MSE\left(\hat{S}^2_{KC2}\right)$ in equation (15) i.e

$$\bullet MSE\left(\hat{S}^2_{CVRP}\right) \le MSE\left(\hat{S}^2_{KC2}\right)$$

$$\Rightarrow \frac{1-f}{n} S_y^4 \left[\lambda_1^2 + (\theta_{40}-1) + (1-2\lambda_1)^2 \gamma(\theta_{04}-1)\right] \le \frac{1-f}{n} S_y^4 \left\{(\theta_{04}-1) + R^2_{KC2}(\theta_{04}-1) - 2R_{KC2}(\theta_{22}-1)\right\}$$

$$MSE\left(\hat{S}^2_{KC2}\right) - MSE\left(\hat{S}^2_{CVRP}\right) = \frac{1-f}{n} S_y^4 \left[\lambda_1^2 + \left((1-2\lambda_1)^2 \gamma - R^2_{KC2}\right)(\theta_{04}-1) - 2R_{KC2}(\theta_{22}-1)\right] > 0$$

4) The MSE of the Proposed $MSE\left(\hat{S}^2_{CVRP}\right)$ estimator is less than the MSE of the Subramani and Kumarapandiyan estimator $MSE\left(\hat{S}^2_{SK}\right)$ in equation (18) i.e

$$\bullet MSE\left(\hat{S}^2_{CVRP}\right) \le MSE\left(\hat{S}^2_{SK}\right)$$

$$\Rightarrow \frac{1-f}{n} S_y^4 \left[\lambda_1^2 + (\theta_{40}-1) + (1-2\lambda_1)^2 \gamma(\theta_{04}-1)\right] \le \frac{1-f}{n} S_y^4 \left\{(\theta_{04}-1) + R^2_{SK}(\theta_{04}-1) - 2R_{SK}(\theta_{22}-1)\right\}$$

$$MSE\left(\hat{S}^2_{SK}\right) - MSE\left(\hat{S}^2_{CVPR}\right) = \frac{1-f}{n} S_y^4 \left[\lambda_1^2 + \left((1-2\lambda_1)^2 \gamma - R^2_{SK}\right)(\theta_{04}-1) - 2R_{SK}(\theta_{22}-1)\right] > 0$$

4. NUMERICAL COMPARISONS

Dataset: The populations considered in this study are natural population datasets. The population is taken from Subramani and Kumarapandiyan (2012) given in page 150. Table 1 below summarized the dataset for the study.

Table 1: Summary of the Dataset

Parameters	Population 1:	Population 2:
N	103	49
n	40	20
$\overline{Y}$	626.2123	116.1633
$\overline{X}$	557.1909	98.6765
ρ	0.9936	0.6904
C_y	1.4588	0.8508
C_x	1.4683	1.0435
S_x	81.8111	102.9709
S_y	91.3549	98.8286
θ_{04}	37.3216	5.9878
θ_{40}	37.1279	4.9245
θ_{03}	17.4683	1.5200
θ_{30}	16.9128	1.1128
θ_{22}	37.2055	4.6977
M_d	308.0500	64.0000

δ	0.2280	0.6984
γ	0.7959	0.6267
λ_1	1.4856	1.2692

Therefore, we see the merit of the proposed estimator $\hat{S}^2_{CVRP}$ over some existing Ratio estimators $\hat{S}^2_I$ (Isaki's ratio estimator), $\hat{S}^2_{KC1}$ (kadilar and Cingi's ratio estimator), $\hat{S}^2_{KC2}$ (Kadilar and Cingi's ratio estimator) and $\hat{S}^2_{SK}$ (Subramani and Kumarapandiyan's ratio estimator) for simple random sampling in Table 2.

Table 2: The MSE of $\hat{S}^2_I$, $\hat{S}^2_{KC1}$, $\hat{S}^2_{KC2}$ and $\hat{S}^2_{SK}$ and the proposed estimator $\hat{S}^2_{CVRP}$

		Populations	
		1	2
Estimators	$\hat{S}^2_I$	670393270	7235508
	$\hat{S}^2_{KC1}$	670169790	7228570
	$\hat{S}^2_{KC2}$	670384403	7234298
	$\hat{S}^2_{SK}$	668667061	7228859
	$\hat{S}^2_{CVRP}$	150534949	1187207

Table 3: Percentage relative efficiency of different estimators with respect to $\hat{S}^2_I$

		Populations	
		1	2
Estimators	$\hat{S}^2_I$	100	100
	$\hat{S}^2_{KC1}$	100.0334	100.0950
	$\hat{S}^2_{KC2}$	100.0013	101.0167
	$\hat{S}^2_{SK}$	100.2582	100.0920
	$\hat{S}^2_{CVRP}$	445.3406	609.4563

4.1. **Discussion of Result**

The main focus of this study was to propose more efficient estimator based on simple random sampling. We have proposed a ratio-product estimator and obtained its asymptotically optimum estimator (AOE) with its approximate MSE formula in simple random sampling.

Theoretically, we have demonstrated that the proposed estimator was more efficient than other estimators, $\hat{S}^2_I$, $\hat{S}^2_{KC1}$, $\hat{S}^2_{KC2}$ and $\hat{S}^2_{SK}$ under their optimum value.

In addition, we support these theoretical results numerically using the data sets as shown in Table 1. Table 2 provides the MSE of the proposed estimator ($\hat{S}^2_{CVRP}$) and some existing estimators, $\hat{S}^2_I, \hat{S}^2_{KC1}$, $\hat{S}^2_{KC2}$ and $\hat{S}^2_{SK}$ as far as mean squared error criterion is concerned. Table 3 provides that there is a considerable gain in efficiency by using proposed estimator over the estimators $\hat{S}^2_I, \hat{S}^2_{KC1}, \hat{S}^2_{KC2}$ and $\hat{S}^2_{SK}$. This shows that even if the scalar λ_1 deviates from its optimum values, the suggested estimator $\hat{S}^2_{CVRP}$ will yield better estimates than, $\hat{S}^2_I, \hat{S}^2_{KC1}, \hat{S}^2_{KC2}$ and $\hat{S}^2_{SK}$.

5. CONCLUSION

The main focus of this study was to propose more efficient estimator based on simple random sampling. Evidence from the study revealed that the proposed estimator is more efficient than the already existing ratio type estimators based on some certain efficiency conditions.
Hence, we conclude that the proposed estimator is more efficient than the other estimators in case of their optimality in simple random sampling. Thus, it is preferred to use the proposed estimator in practice over the existing estimators considered.

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