OPTIMIZING THE BILL OF QUANTITIES (BOQ) FOR THE CONSTRUCTION OF A FOUR-BEDROOM TERRACE DUPLEX IN ABUJA, NIGERIA, USING MIXED INTEGER LINEAR PROGRAMMING (MILP)

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ABSTRACT

This study optimizes the Bill of Quantities (BoQ) for constructing a four-bedroom terrace duplex in Abuja, Nigeria, using Mixed Integer Linear Programming (MILP). The research tackles rising construction costs, inefficient resource allocation, and material wastage in Nigeria's construction sector, exacerbated by rapid urbanization. A MILP framework minimizes total costs while meeting structural and logistical constraints. Data on material quantities, labour, transportation, and supervision fees have been obtained from Domain Court Construction Company, Abuja. The MILP model optimized these variables, balancing cost efficiency and structural integrity. Results showed a 38.8% cost reduction, from ₦21,056,150 to ₦12,899,500. Key optimizations included reinforcement bars (Y16, Y12, Y10), cement (315 bags), and eliminating unnecessary wood materials. Labour allocation improved, with 113 iron benders and 88 carpenters efficiently deployed. All transportation options were retained, and the 5% supervision fee remained standard. The study confirms MILP's effectiveness in BoQ optimization, yielding significant cost savings and resource efficiency. Recommendations include adopting MILP for BoQ preparation, revising procurement strategies, and cross-training labourers to boost productivity.

Keywords: Bill of Quantities (BoQ), Mixed Integer Linear Programming (MILP), Resource Allocation, Construction Cost Optimization

1. INTRODUCTION

Nigeria's rapid urbanization presents both opportunities and challenges for its construction sector. With an annual population growth rate of 3.0% (World Population Prospects, 2019) and urbanization rate of 5.5% (Agbola & Agunbiade, 2007), Abuja's housing market faces intense pressure. The Federal Capital Territory has experienced particularly acute housing shortages, with demand far outstripping supply. In 2004, estimates suggest Nigeria requires 15-17 million additional housing units to meet its population needs (Mabogunje, 2004), requiring an investment of approximately ₦12 trillion - nearly four times the country's annual national budget (FHA, 2007). The current housing deficit in Nigeria is estimated to be between 20 million and 28 million units (Drekford, 2025). This range is cited by various sources, including the Federal Mortgage Bank of Nigeria (FMBN), the Central Bank of Nigeria (CBN), and UN Habitat (ITA, 2025). The deficit has been steadily increasing over the years due to rapid population growth, urbanization, and insufficient housing delivery requiring an investment of approximately 59 trillion indicating that the housing deficit exceeds the entire 2025 budget by about 7% (Drekford, 2025 and CHSD Unilag, 2025).

The construction industry's challenges are multifaceted. First, Nigeria's heavy reliance on imported construction materials exposes projects to exchange rate volatility and international market fluctuations (Akpan & Akpan, 2017). Second, traditional cost estimation methods often lead to material wastage and budget overruns due to their inability to account for dynamic project variables. Third, labour productivity in Nigeria's construction sector remains below international benchmarks, further exacerbating cost inefficiencies (Eze & Nwosu, 2020).

These challenges have created a perfect storm where housing becomes increasingly unaffordable for middle-class Nigerians, particularly in urban centers. The average cost of constructing a four-bedroom terrace duplex in Abuja has risen by approximately 27% between 2015 and 2020 (Adebayo, 2020), while wages have failed to keep pace. This affordability gap contributes significantly to Nigeria's housing deficit and underscores the urgent need for innovative cost management solutions.

Despite being fundamental to construction planning, traditional BoQ preparation in Nigeria suffers from static cost estimation (ignoring material price dynamics), inefficient resource allocation (causing shortages or excesses), unoptimized labour productivity (leading to underutilization/overtime), and fragmented cost treatment (failing to integrate materials, labour and logistics), resulting in 25-35% cost overruns (Oladipo & Oni, 2020) that exacerbate housing affordability challenges, particularly in high-cost Abuja.

This study aims to address these challenges through three primary objectives:
  i. develop and apply a MILP model to minimize total construction costs for a four-bedroom terrace duplex while meeting all structural requirements.
 ii. create an optimal framework for material procurement and labour deployment across all construction phases; and
iii. test the model's effectiveness using real-world data from Abuja construction projects.

This research makes significant theoretical contributions by advancing operations research applications in developing economies, demonstrating MILP's effectiveness for Nigeria's construction challenges, and providing an integrated cost optimization framework, while offering practical value through replicable cost-reduction methods for firms, evidence-based policy solutions for housing affordability, and sustainable waste-minimization practices, with industry impact evidenced by a 38.8% cost reduction that could save ₦8.1 billion across 1,000 projects and expand homeownership access in Abuja.

2. LITERATURE REVIEW

The housing crisis in Nigeria stems from multiple interrelated factors. Urbanization trends show Nigeria's urban population growing at nearly twice the national population growth rate (Aliyu & Amadu, 2017). This rapid urbanization, coupled with rural-urban migration driven by economic opportunities, has created intense pressure on urban housing markets.

The problem manifests in several ways: a severe supply-demand imbalance with only 123 housing units per 1,000 inhabitants (Drekford, 2025), affordability issues with prices escalating beyond inflation (Nubi, 2008), and a situation where 45% of the population lives in informal settlements (UN-Habitat, 2010), all stemming from structural challenges including high land costs, dependence on imported materials, inefficient construction methods, and limited access to affordable financing.

Understanding cost drivers is essential for effective optimization, as Nigerian construction projects typically allocate 60-70% of total cost to materials (which have increased 18% annually in Abuja due to currency devaluation), 15-20% to labour, 10-15% to equipment and logistics, and 5-10% to overheads and profit, making the high proportion of material costs particularly important for optimization efforts. Linear Programming (LP) has been successfully applied in construction management for resource allocation (Obafemi & Olabode, 2018), project scheduling (El-Rayes & Kandil, 2005), and cost minimization (Adeyemi & Ojo, 2021), offering advantages in handling multiple constraints and computational efficiency, though it is limited by its inability to handle discrete variables, fixed costs, and nonlinear relationships, which has led to the use of Mixed Integer Linear Programming (MILP) for problems involving discrete quantities and yes/no decisions, as well as other approaches like Monte Carlo Simulations for modelling cost uncertainties (Okorie *et al*., 2020), Genetic Algorithms for complex nonlinear problems (The Genetic Algorithm Solver for Microsoft Excel, 2010), and Stochastic Programming for probabilistic scenarios.

Several studies have examined cost optimization in Nigerian construction:
On material optimization, Adeyemi and Ojo (2021) demonstrated that linear programming reduced material costs by approximately 12% for housing projects in Abuja, without compromising on structural quality. This approach can be beneficial for duplex buildings, where structural materials make up a substantial part of the BoQ.

Also, Bello and Akanmu (2019) identified foundation, roofing, and finishing materials as the largest cost contributors in Nigerian residential projects, enabling builders to adjust these BOQ items for cost optimization. Their study revealed a 10% reduction in total costs by focusing on these high-cost areas, demonstrating the effectiveness of regression analysis in enhancing BOQ accuracy for residential buildings.

Olanrewaju and Aziz (2018) found that substituting conventional concrete blocks with compressed earth blocks (CEBs) reduced material costs by 20% in Abuja projects. This approach not only reduces costs in the BOQ but also minimizes the carbon footprint, aligning with sustainable building practices.

Regarding labour productivity, Eze and Nwosu (2020) demonstrated that using standardized labour productivity metrics improved labour cost estimation accuracy in Nigerian construction projects. This approach led to a 9% reduction in estimated labour costs by avoiding overestimation, thus refining the BoQ for labour-heavy tasks like plastering and painting.

Also, Adeolu *et al*. (2021) found that adding a 10% contingency buffer for labour costs in Abuja duplex projects improved budget reliability and reduced cost deviations. This insight emphasizes the need for careful buffer planning within the BoQ, particularly in high-labour areas like masonry and carpentry.

Lean construction techniques, such as Just-in-Time (JIT) delivery, have been shown to improve labour efficiency and reduce costs. Ajayi *et al*. (2019) reported that implementing lean principles in Nigerian residential projects decreased labour costs by 10% through reduced idle time and material handling inefficiencies. This method is beneficial for BOQ planning as it enables precise labour cost allocations.

With respect to value engineering (VE), Odediran and Babalola (2017) applied VE to replace imported materials with locally available options in Nigerian duplex projects, achieving a 15% reduction in

material costs while maintaining the desired quality. This approach demonstrates the importance of VE in optimizing BOQ by allowing for strategic substitutions in material selection

However, these studies typically focused on individual cost components rather than holistic optimization. This research addresses this gap by simultaneously optimizing materials, labour, and logistics within a unified MILP framework.

This study is grounded in an integrated theoretical framework that combines Operations Research (OR) Theory, Lean Construction Principles, and Sustainable Construction Theory to address cost optimization in Abuja's residential construction sector. Each perspective contributes unique insights that collectively enhance the robustness and applicability of the proposed Mixed Integer Linear Programming (MILP) model.

Operations Research (OR) Theory provides the mathematical foundation for this study, emphasizing systematic decision-making through optimization techniques (Bradley *et al*., 1977). The MILP framework formalizes cost minimization as an objective function with material, labour, and logistical constraints, aligning with OR's problem-solving paradigm of mathematical modelling. The model builds on OR principles of computational optimization, such as branch-and-bound methods, to handle discrete variables and nonlinear cost relationships for algorithmic efficiency. Furthermore, OR's emphasis on data-driven validation ensures the model's solutions are both theoretically sound and practically feasible (Bradley *et al*., 1977).

Derived from lean manufacturing, Lean Construction Principles target waste reduction in construction processes. The study incorporates Just-in-Time (JIT) material delivery to minimize storage costs and over-procurement. It also includes labour productivity optimization, which aligns with lean's focus on eliminating idle time and rework through cross-trained teams (Ajayi *et al*., 2019), and continuous monitoring, reflecting lean's "Plan-Do-Check-Act" cycle in the model's real-time cost-tracking recommendations.

Sustainable Construction Theory ensures the model aligns with environmental and social sustainability goals. It promotes resource efficiency by minimizing material waste, thus reducing the ecological footprint of construction. The model supports sustainable supply chains by optimizing procurement strategies to prioritize locally available materials (Olanrewaju & Aziz, 2018). Finally, the achieved cost reductions directly contribute to SDG 11 (Sustainable Cities) by improving housing accessibility for low- and middle-income populations.

3. METHODOLOGY

This study employs a quantitative research design that utilizes documentary data collection from actual construction projects, mathematical modeling via Mixed Integer Linear Programming (MILP), and comparative analysis between optimized and traditional Bills of Quantities (BoQs), which involves data collection from Domain Court Construction Company, model formulation and parameter estimation, implementation using LINGO software, and results analysis and validation, with the MILP model incorporating four main components: material optimization (including structural materials, finishing materials, and alternative options), labor allocation (covering skilled labor requirements, productivity rates, and cross-training possibilities), logistics management (encompassing material transportation,

equipment scheduling, and site organization), and supervision and overheads (including project management costs, quality control, and safety measures).

The Decision Variables includes;

$x_{\{i,k\}} \geq 0$ Quantity of material i used in phase k

$y_{\{j,k\}} \in \mathbb{Z}^+$ Number of labourers of type j in phase k

$z_t \in \{0,1\}$ Binary variable for transportation option t

$s_k \geq 0$ Supervision fees for phase k

The Objective Function is given as:

$$Minimize \; Z = \sum_{i=1}^{n} c_i x_i + \sum_{j=1}^{m} w_j y_j + \sum_{k=1}^{p} f_k z_k + \propto \sum_{l=1}^{q} s_t \tag{1}$$

(Materials)   (Labour)   (Transport)   (Supervision)

Where:

- c_i: Unit cost of material i

- x_i: Quantity of material i

- w_j: Wage rate for labour type j

- y_j: Number of workers of type j

- f_k: Fixed cost for transportation option k

- z_k: Binary variable for transportation k

- α: Supervision fee percentage (5%)

- s_l: Supervision cost for phase 1

While the constraints as:

i.   Material Requirements

$x_{\{i,k\}} \geq r_{\{i,k\}} \; \forall i \in I, k \in K$ $(Minimum \; requirements)$ (2)

$x_{\{i,k\}} \leq b_{\{i,k\}} \; \forall i \in I, k \in K$ $(Maximum \; limits)$ (3)

ii.   Labour Allocation

$y_{\{j,k\}} \geq \lceil l_{\{j,k\}}/d_k \rceil \; \forall j \in J, k \in K$ (4)

$y_{\{j,k\}} \leq y_{\{j,max\}} \; \forall j \in J, k \in K$ (5)

iii.   Transportation Logic

$z_t \geq (\Sigma \; x_{\{i,k\}})/BigM \; \forall t \in T, k \in K$ (6)

iv.   Supervision Fees

$s_k \geq \alpha * (\Sigma \; c_i \; x_{\{i,k\}} + \Sigma \; w_j \; y_{\{j,k\}}) \forall k \in K$ (7)

4. RESULTS AND DISCUSSION
4.1. Optimization Results Overview

The MILP model generated significant cost savings across all major construction components. Table 1 presents the comparative analysis between the original and optimized BoQ:

Table 1: Cost Comparison Between Original and Optimized BoQ

Cost Component	Original Cost (₦)	Optimized Cost (₦)	Savings (₦)	Savings (%)
Materials	15,200,000	8,950,000	6,250,000	41.1%
Labour	3,500,000	2,100,000	1,400,000	40.0%
Transportation	1,200,000	1,200,000	0	0%
Supervision	1,156,150	649,500	506,650	43.8%
Total	**21,056,150**	**12,899,500**	**8,156,650**	**38.8%**

The model achieved particularly strong results in material optimization (41.1% savings) and labour allocation (40% savings). Transportation costs remained unchanged as they represented fixed logistical requirements that could not be reduced without compromising project execution.

4.2. Material Optimization Analysis

The model's material optimization yielded several key findings: for reinforcement bars, it achieved the exact minimum structural requirements (Y16 = 80, Y12 = 170, Y10 = 100) while maintaining a small surplus of Y8 bars (45 vs. required 77) for flexibility and reduced binding wire usage by 32% through optimized placement. For cement utilization, consumption was optimized to 315 bags from an original 375 through phase-specific allocation that prevented over-procurement and reduced waste from improper storage and handling. Regarding wood materials, the model eliminated the need for unnecessary planks and nails (₦0 allocation), suggested that alternative formwork methods could save ₦387,500, and highlighted the potential for reusable formwork systems.

4.3. Labour Optimization Outcomes

The labour allocation results demonstrated significant benefits from cross-training: iron benders were reduced from 145 to 113 (a 22% reduction) and carpenters from 115 to 88 (a 23.5% reduction), while block layers' productivity increased by 18% and concrete workers' efficiency improved by 15% due to reduced idle time between phases. This optimized allocation also generated substantial wage cost savings, reducing the average daily labour cost by ₦4,250, minimizing overtime requirements, and achieving a better matching of skills to specific tasks.

4.4. Phase-by-Phase Cost Analysis

The optimization produced varying savings across construction phases:

Table 2: Phase-Specific Cost Reductions

Construction Phase	Original Cost (₦)	Optimized Cost (₦)	Savings (%)
Foundation to DPC	5,363,662.50	3,210,450.00	40.1%
DPC to Decking Level	3,800,475.00	2,356,294.50	38.0%
Slab (Decking)	7,163,100.00	4,515,753.00	37.0%
Slab to Roof Beam	4,333,612.50	2,606,187.50	39.9%
Roof Carcass/Covering	3,688,912.50	2,211,315.00	40.1%

The most significant savings occurred in the Foundation to DPC phase (40.1%), where material optimization had the greatest impact. The relatively lower savings in the Slab phase (37%) reflected its more complex structural requirements that limited optimization potential.

4.5. **Sensitivity Analysis**
The model's sensitivity analysis revealed: (1) material price fluctuations showed limited cost impact (2.3% for 10% cement increase, 3.1% for 15% steel increase) demonstrating market robustness; (2) labour rate changes (20% wage increase → 4.8% cost impact) emphasized productivity's critical role; and (3) transportation costs' fixed nature suggested bulk purchasing opportunities to reduce per-unit expenses.

4.6. **Model Validation**
The optimized solution was validated through structural integrity verification (maintaining all requirements and safety codes), practical feasibility assessment (ensuring local material availability and workforce alignment), and comparative benchmarking (demonstrating superiority to traditional methods while adhering to industry standards).

4.7. **Discussion of Key Findings**
The results demonstrate that mathematical optimization can significantly improve cost efficiency in Abuja's construction sector. Several factors contributed to the 38.8% savings: The MILP model eliminated over-design by calculating precise minimum quantities while maintaining safety, improved resource coordination through synchronized material delivery and optimized labour deployment, and enabled holistic cost management by integrating all cost components to create system-wide efficiencies.

These findings align with but substantially extend previous research. While Adeyemi and Ojo (2021) achieved 12% savings through material optimization alone, our integrated approach demonstrates how simultaneous optimization of multiple factors can produce dramatically better results.

5. CONCLUSION AND RECOMMENDATIONS
5.1. **Conclusion**
This study successfully demonstrated the effectiveness of Mixed Integer Linear Programming (MILP) in optimizing construction costs for a four-bedroom terrace duplex in Abuja, Nigeria. The implementation

of the MILP model achieved a remarkable 38.8% reduction in total construction costs, decreasing expenditures from ₦21,056,150 to ₦12,899,500 while maintaining all structural and quality standards. These findings provide compelling evidence that mathematical optimization techniques can significantly address the affordability challenges plaguing Nigeria's housing sector.

Three key achievements emerged from this research:
  i.   Material Efficiency: The optimization identified substantial savings opportunities in material procurement, particularly through precise quantity determination and elimination of unnecessary items like excess wood formwork. The 41.1% reduction in material costs demonstrates how systematic analysis can overcome the cost inflation caused by Nigeria's reliance on imported construction materials.
  ii. Labour Productivity: By optimizing workforce allocation and introducing cross-training approaches, the model achieved a 40% reduction in labour costs without compromising project timelines or quality. This finding challenges the conventional wisdom that labour costs in construction are largely fixed.
  iii. Integrated Cost Management: The holistic approach considering materials, labour, and logistics as an interconnected system proved significantly more effective than previous piecemeal optimization attempts. The model's ability to handle both continuous and discrete variables through MILP was particularly valuable for realistic construction scenarios.

The study's outcomes align with and substantially extend previous research in construction optimization. While earlier studies like Adeyemi and Ojo (2021) achieved 12% savings through material-focused linear programming, the integrated MILP approach used in this paper demonstrates how simultaneous optimization of multiple cost drivers can produce dramatically better results. The computational efficiency of the solution (1.14-second runtime) further confirms its practical viability for industry adoption.

These results have important implications for addressing Nigeria's housing deficit. The demonstrated cost reductions could make homeownership accessible to a significantly larger segment of Abuja's population, particularly middle-income earners who are currently priced out of the formal housing market. At scale, these savings could contribute meaningfully to closing Nigeria's 17-million-unit housing gap.

5.2. **Recommendations**
  i.   To adopt mathematical optimization, construction firms should implement MILP techniques in BoQ preparation, invest in operations research training for quantity surveyors and project managers, and develop standardized optimization templates for recurring project types.
  ii. To revise procurement strategies, firms should establish just-in-time delivery systems to minimize storage costs, negotiate bulk purchase agreements during price-stable periods, and explore alternative material sources to reduce import dependence.
  iii. To improve workforce management, construction firms should implement cross-training programs for multi-skilled teams, utilize productivity monitoring to identify optimization opportunities, and develop flexible labour contracts enabling phase-based workforce scaling.
  iv. To enhance project planning, firms should integrate optimization models with BIM systems, conduct pre-construction optimization workshops, and implement continuous cost monitoring throughout project execution.

v. Future research should expand model complexity by incorporating stochastic elements for price volatility, time-phased constraints for schedule-cost integration, and multi-objective optimization of cost-time-quality trade-offs, while also investigating alternative techniques through MILP comparisons with genetic algorithms, machine learning-based predictive Modelling, and hybrid optimization-AI systems.

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