Ahmadu Bello University, Zaria

Department of Mathematics

2013/2014 First Semester Examination

**COSC 401 Introduction to Algorithms & Complexity Analysis**

**Date**: June 2014 **Time Allowed**: 120 Minutes

**Instructions**:

1. Attempt ANY FOUR questions.
2. Write all your answers in the spaces provided on this Question Paper.

**Student’s Registration Number**:……………………………………………… **Signature**:………………………………

**Date of Examination**:………………………………………………………………**Time**:………………………………………

**Scores**:

|  |  |  |
| --- | --- | --- |
| **Question** | **Maximum Scores** | **Scores Obtained** |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| 5 | 20 |  |
| 6 | 20 |  |
| **Total** | **80** |  |

1. (**20 Marks**). Answer both questions
	1. (**10 Marks**). Determine the number of character comparisons that will be made by the brute-force algorithm in searching for the pattern GOODIE in the text

WHAT\_IS\_GOOD\_FOR\_THE\_GOOSE\_IS\_GOOD\_FOR\_THE\_GANDER

* 1. **(10 marks)**. The Quicksort algorithm works based on a split (i.e., partition) phase and a merge phase. Given the sequence:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 7 | 6 | 9 | 12 | 5 | 7 | 6 | 4 | 7 | 4 | 11 | 12 | 6 | 8 | 9 |

Assuming the first number, 3, as the pivot, trace the partition phase of this algorithm. You should indicate the numbers swapped in each stage.

**Answer:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. Arrangement of numbers after Step 1 is as follows. Numbers swapped are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. Arrangement of numbers after Step 2 is as follows. Numbers swapped are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. Arrangement of numbers after Step 3 is as follows. Numbers swapped are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. Arrangement of numbers after Step 4 is as follows. Numbers swapped are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

 |

1. **(20 marks)**. There are four people who need to be assigned to four jobs, one person per job. The cost of assigning person **i** to job **j** is **C[i,j]**. Find an assignment that minimizes the total cost. You must use the brute-force approach to your solution to generate all legitimate assignments, compute their costs, and select the cheapest one.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Job 0 | Job 1 | Job 2 | Job 3 |
| Person 0 | 9 | 2 | 7 | 8 |
| Person 1 | 6 | 4 | 3 | 7 |
| Person 2 | 5 | 8 | 1 | 8 |
| Person 3 | 7 | 6 | 9 | 4 |

Solution

1. **(20 marks)**. Attempt both parts of this question:
	1. **(10 marks)**. Let A and B be large n-digit integers. Show that the number of one-digit multiplications required to calculate A\*B using a divide-and-conquer algorithm is $n^{log\_{2}3}$.

**Hint**: $A\*B=A\_{1}\*B\_{1}.10^{n}+\left(A\_{1}\*B\_{2}.+A\_{2}\*B\_{1}\right).10^{\frac{n}{2}}$.$+ A\_{2}\*B\_{2}$, where A1,A2,B1,B2 are n/2-digt integers

* 1. **(5 marks).** Compute A \* B where A = 2135 and B = 4014 using the divide-and-conquer algorithm referred to in (a) above.
	2. **(5 marks).** How many one-digit multiplications should be performed in (b) above based on this algorithm. Explain your answer.
1. (**20 Marks**). Attempt both the following questions:
	1. **(10 marks)**. There is a pile of 121 chips. Two players, Ali and Isa, take turn by removing from the pile at least 1 chip and at most 7 chips. The number of chips taken can vary from move to move. The winner is the player that takes the last chip. Assuming that Ali will be the first to move, write down[[1]](#footnote-1) the
		1. minimum sequence of moves in which Ali emerges winner. How many chips will Ali take during his last move?
		2. maximum sequence of moves in which Ali emerges winner. How many chips will Ali take during his last move?
	2. **(10 marks)**. Consider a binary T with n nodes. Show that the inorder traversal of T is in $θ(n)$. Explain your answer.
2. **(20 marks).** Answer both the following questions:
	1. **(10 marks).** Generate all permutations of {1, 2, 3, 4} bythe lexicographic-order algorithm
	2. **(10 marks).** For the Josephus problem, compute J(n) by filling the table below:

|  |  |
| --- | --- |
| n | Josephus number, J(n) |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |

1. **(20 marks).** Use the following graph to answer the questions below.



Complete the following table by writing the order of arranging the vertices according to the traversal/algorithm indicated. You **must** arrange the vertices of the graph in sorted order (i.e., dictionary order) as much as possible.

1. (2.**5 Marks**). Depth-first traversal (from vertex **a**)
2. (2.**5 Marks**). Breadth-first traversal (from vertex **a**)
3. **(5 Marks).** Topological ordering of vertices (from the appropriate vertex) using the source removal algorithm.
4. (**10 Marks**). Determine the order of growth for solution of the recurrence: T(n)=4T(n/2)+1, T(1)=1, using backward substitution.
1. If there are no moves possible for these questions, explain why. [↑](#footnote-ref-1)